

An evidence-based trajectory of spatial reasoning development for 7- to 11-year-olds.

Spatial reasoning involves understanding and visualising spatial relations and the spatial properties of objects, including spatial aspects of quantities. Spatialising the mathematics curriculum by emphasising thinking and working spatially has broad benefits for mathematics, including geometry, measures, number, algebra and statistics. Research shows that teaching children to think and work spatially results in substantially improved mathematics performance, with lifelong benefits.

This trajectory of spatial reasoning development for 7- to 11-year-olds complements the trajectory of spatial reasoning development for birth to seven years developed by the Early Childhood Maths Group (see [ECMG Spatial Reasoning Toolkit](#)). These two trajectories are informed by extensive review of research in this area and together offer activities for children from birth to 11 years. This trajectory is presented in four sections, covering space (spatial relations) and shape (objects and properties) for 7- to 9-year-olds and 9- to 11-year-olds. Three columns identify the areas of spatial reasoning, the progression in children's learning and developmentally appropriate activities. Each child develops in their own unique way: age bands are approximate and dependent on previous experience, so should be used as a guide rather than age-related expectations. Spatial reasoning is wider than traditional geometry, including aspects of space and shape such as position and direction, navigation, perspective-taking, scaling, transformations, shape properties and structure, composition and decomposition; the document could be used to substitute a geometry curriculum, with activities integrated throughout the curriculum.

This document forms part of a series of Royal Society publications on spatial reasoning which can be found at: [Royal Society Curriculum and Assessment](#) and accompanies our earlier work on the [ECMG Spatial Reasoning Toolkit](#).

The series includes:

- [RS ACME Primary and early years expert panel perspective: Spatial reasoning - June 2024 \(royalsociety.org\)](#)
- [Progression of Spatial Reasoning across age bands](#) (Appendix 1 of document listed above)
- This document, *An evidence-based trajectory of spatial reasoning development for 7- to 11-year-olds*.
- Whole-school approach case studies: *Supporting Primary Children's Spatial Reasoning in Geography* and *Design & Technology*

For earlier stages of spatial reasoning development, see the [ECMG trajectory of spatial reasoning development for birth to seven years](#)

Contact details: s.gifford@roehampton.ac.uk

Acknowledgements: This document was written by Dr Sue Gifford¹, Prof. Emily Farran² and Dr Alison Borthwick¹ with contributions from the Joint Primary ATM/MA group, Clare Christie (mathematics lead teacher) and Remke Klapwijk (Delft University of Technology).

¹Royal Society ACME Primary and Early Years Expert Panel

²University of Surrey, UK

To cite this document: Gifford, S., Farran, E.K., Borthwick, A. (2024). *An evidenced-based trajectory of spatial reasoning development for 7- to 11-year-olds*. <https://royalsociety.org/-/media/policy/projects/mathematics-education/spatial-reasoning-7-11-year-olds.pdf> https://doi.org/10.31234/osf.io/394nk_v3

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Spatial Reasoning involves:

Space (spatial relations)	Shape (objects and properties)
• Position – <i>Where?</i> In relation to one or two things, e.g. <i>next to, between, in front of, behind, relative to the viewer</i>, and using coordinates. • Direction – <i>Which way?</i> Moving around, e.g. <i>over, under, forwards/backwards, left, right</i>. • Dimensions – Distance, e.g. <i>How far away?</i> Area, e.g. <i>near, in the middle</i>. • Transformations – Rotation (<i>turning</i>), translation (<i>sliding</i>) and reflection (<i>flipping</i>), e.g. moving a shape or jigsaw puzzle piece to fit or match. • Perspective-taking – Appearance from different viewpoints: ○ Visibility – <i>what</i> can be seen, e.g. hidden or partially visible. ○ Position – <i>where</i> objects are in relation to each other, e.g. things behind each other appear to overlap. ○ Orientation – <i>which way up?</i> e.g. <i>upside down, back to front, tipped over</i>. ○ Appearance – e.g. <i>how</i> circles can look like ovals from certain viewpoints. ○ Scaling – zooming in and out e.g. small-world play (toy farms), reading maps. • Navigation – e.g. wayfinding, map-making and routes.	• Identifying – <i>What?</i> 2D and 3D objects (regular and irregular) such as cups, clothes, jigsaw pieces, leaves and clouds, e.g. <i>circle, rectangle, triangle, heart-shaped, cuboid, cone, ball, roof-shaped</i>. • Properties including: ○ Size, length, area, capacity and volume, e.g. <i>big, tall, wide; Will it fit in?</i> ○ Sides, faces, edges, lines, radius, e.g. <i>straight/ curved, wiggly, zig-zag; equal and unequal</i>. ○ Corners, angles, e.g. <i>points, vertices, right angle, square corner, sharp, obtuse</i>. ○ Symmetry – in 2D and 3D, reflective and rotational. • Composing and Decomposing ○ Composing – fitting together 2D and 3D shapes, using interrelationships between properties, angles, e.g. with jigsaw puzzle pieces, pattern blocks, nesting containers and construction. ○ Decomposing – disembedding (parts within wholes), cutting and folding, cross-sections, 2D and 3D, e.g. unfolding boxes to make nets and recomposing, <i>Which net would make a cube?</i> • Scaling – identifying the same item in different sizes, similarity, proportions, enlarging and shrinking, <i>doubling</i>.

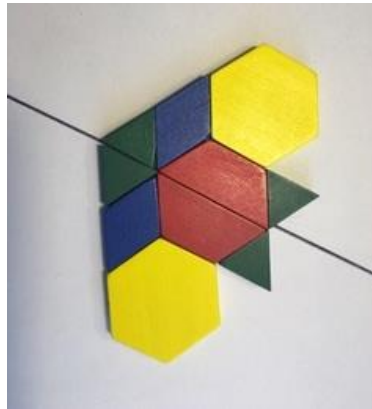
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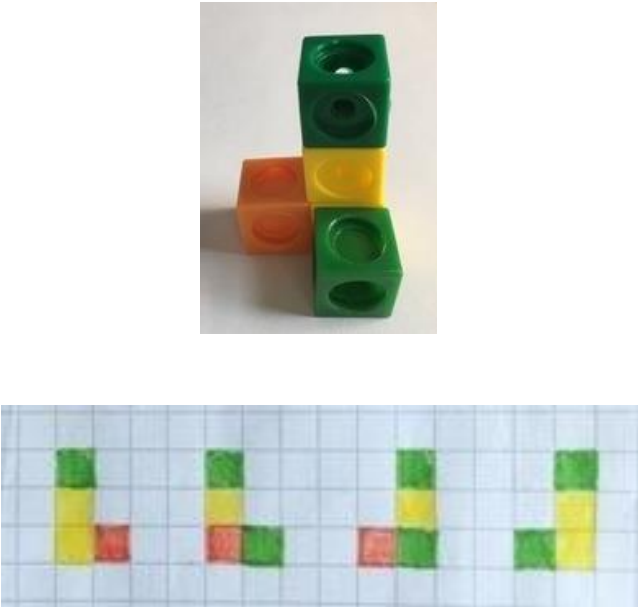
Space (spatial relations)		
Lower KS2 (7 to 9 years)		
	Children are learning to	Activities to support learning
Position Where? in relation to one or two things e.g. next to, between; relative to the viewer, e.g. in front of, behind, using coordinates.	Continue to use the language of position, <i>horizontal</i>, <i>vertical</i>, and supported by gestures. Identify relative positions on a line and 2D positions within an area, using terms such as <i>middle</i>, <i>midpoint</i>, <i>nearer</i>, <i>edge</i>, and simple coordinates.	Number lines Visualise numbers on a number line, using rounding to estimate positions (e.g. the midpoint of a line). <i>Can you imagine where the number 157 goes on a 0-1000 number line? Is it closer to 100 or to 200?</i> Bar charts Interpret the different heights of bars on a bar chart, with explanations (e.g. <i>How many times did you jump up and down in one minute on different days?</i>). <i>Which column is higher? What does this mean?</i> Grids Challenge children to copy arrangements of counters on a grid. https://prek-math-te.stanford.edu/spatial-relations/space-assessment-videos

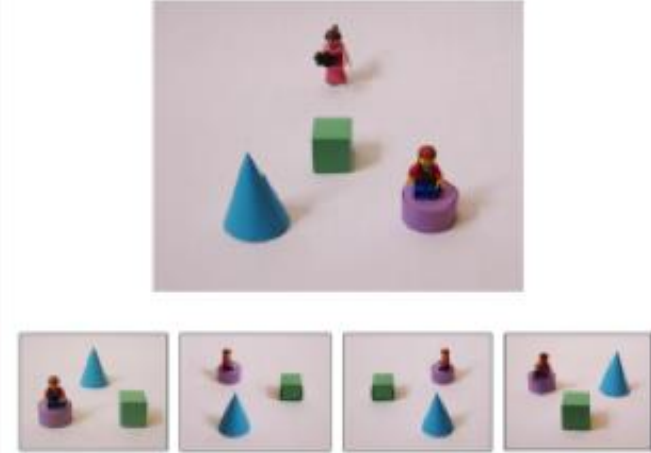
Direction <i>Which way? Moving around, e.g. up/down, forwards/backwards, left, right.</i>	Continue to use the language of direction using appropriate language, <i>left, right, diagonal</i>. Describe direction from the origin, e.g. left 3 and up 5. Identify turns that are more or less than 90 degrees.	Giving and drawing directions Work out a route from one place to another, draw it and then exchange with another group. <i>Did it work?</i> Programmable robots Direct a friend or robot through a maze and predict the turns needed (half, quarter and three-quarter turns). <i>What turns does the robot need to take next, to get to the end? Can you imagine the route in your head?</i> Create a path for a Beebot with 10 cm lengths and then predict the code needed to navigate the path.
Dimensions Distance, e.g. <i>How far away?</i> Area, e.g. <i>near, in the middle.</i>	Estimate distance between places in large-scale space. Use representations to place things at approximately correct relative distances. Continue to use the language of dimension.	Playground maths Find how far 10 metres is, in different places in the playground, using metre sticks and trundle wheels. Develop mental benchmarks for lengths and distances: <i>As long as ... From the gate to ... You know how far one metre is - so how far is 10 metres?</i> Posing challenges: <i>Do you think you can throw a ball 10 metres? Can you estimate the distance that you have thrown the ball, to the nearest metre? Can you measure the distance that you have thrown the ball, to the nearest quarter metre?</i> Relative distances Identify relative distances on the number line: e.g. Is 132 nearer to 100 or 200? Timelines Draw personal timelines with key events, e.g. <i>On your timeline, can you put when your brother was born and when you started school?</i> Use historical timelines and discuss the relative distance in time between events, e.g. the invention of the printing press and the internet.

		Use different sized blocks to illustrate the length of sessions on the daily timetable. If maths starts at 9am, reading starts at 10am and break starts at 10.30am, the block for maths would be twice as tall as for reading (based on Flick & Lakin, 2024). Area Compare irregular shapes by area and use pattern blocks to measure area. Sketches and Visualisation Imagine or sketch the solution to an engineering challenge, such as using cardboard to build a bridge to fit the width of a toy car, before beginning to build (based on Flick & Lakin, 2024).
Transformations Rotation (turning), translation (sliding), or reflection (flipping), e.g. a shape of jig-saw puzzle piece to fit or match	Develop 2D and 3D visualisation skills for movement and rotations (mentally manipulating objects), <i>rotate, translate, reflect, clockwise, anticlockwise</i>, support by using prediction and checking. Reflect a pattern or arrangement over a horizontal or vertical line, progressing to a diagonal line.	Pattern blocks and peg boards Complete the other half of a pattern using reflective symmetry.  Based on Moss et al. (2015)

		Folding and cutting paper <i>How do you fold and cut a strip of paper to create people holding hands? e.g. if you fold the strip in half, in half again and again, will you get the same result as with accordion folding?</i>  https://www.youtube.com/watch?v=kZo68K6vWZ4 Arrays Explore the product of 32, using a 4 x 8 array. <i>Can you transform this array to a different array, for instance a 2 by 16 array?</i>
Perspective-taking Appearance from different viewpoints Visibility - what can be seen, e.g. hidden or partially visible. Position - where objects are in relation to each other, e.g. things behind each other appear to overlap. Orientation - which way up? e.g. Upside down, back to front, tipped over.	Visualise, draw and describe how objects look different from different viewpoints, e.g. overlapping objects, plan view. Interpret and predict what is different between two perspectives.	3D shapes When children are working with a partner using 3D shapes or to build shapes with multilink cubes, ask questions which involve perspective taking: e.g. <i>What does the shape look like to you? How many sides and corners can you see? What can your partner see? Does the shape look the same to them?</i> Scaling <i>Can you draw a map of the playground and place the benches and goalposts in roughly the right places?</i>

Space (spatial relations); 7 to 9 years

Appearance – e.g. how circles can look like ovals from certain viewpoints. Scaling - zooming in and out, e.g. small-world play (toy farms), reading maps.		Different perspectives Draw a 2D representation of a model made with cubes, from four different perspectives.  Based on Mulligan et al. (2020) Hide a small world toy in the environment and tell children what the toy can see: then challenge the child to locate the toy from this.
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		Imagine a scene which somebody else would see from a different viewpoint. <i>Which photograph did the photographer take?</i>  Frick et al. (2014) Bar charts Compare charts using the same data with horizontal versus vertical bars. <i>What is the same and what is different?</i> Research and collect data, make a table and then translate it into a bar chart, e.g. a vote on where to go for the class outing. Compare the table and bar chart: <i>Is one easier to understand?</i> Representing data using 3D material (data physicalisation) Represent numerical data using tangible 3D materials, e.g. represent different quantities with different sized lumps of playdough, balloons hung at different heights or LEGO® bricks spaced at different intervals. Encourage children to visualise how the size and position of objects best show the differences in quantities in their data (based on Zhu et al., 2023).
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Space (spatial relations); 7 to 9 years

Navigation e.g. wayfinding, map-making and routes.	Interpret and draw maps. Plan routes using nearby landmarks. Begin to determine simple shortcuts.	Drawing maps Draw a map of the area around the school or your home. Go outside and visualise a bird's-eye view before drawing the map, or draw a map from memory. Routes and landmarks Explore the spatial relationships between routes and landmarks within an environment (the configural structure of an area), in order to find shortcuts and alternative routes. Map use Use maps with co-ordinates to find 'treasure' (e.g. local maps, Digimap; https://digimapforschools.edina.ac.uk/)
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This table shows the total rainfall and sunshine each year at Heathrow Airport from 2010 to 2015.

Year	Rainfall in mm	Sunshine in hours
2010	521	1,371
2011	509	1,540
2012	700	1,503
2013	560	1,452
2014	864	1,669
2015	562	1,508

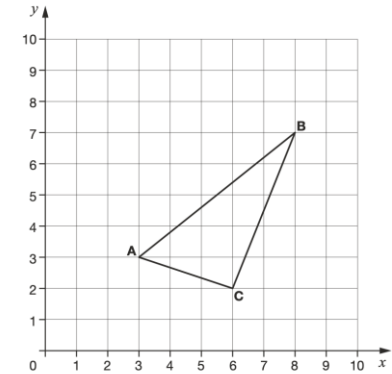
Use this table to complete the graph.

Use a ruler.

Rainfall at Heathrow Airport

Year	Rainfall in mm
2010	521
2011	509
2012	700
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2014	864
2015	562

Key stage 2 tests (2023)

		Coordinates Identify the coordinates of a point on a graph:  ABC is a triangle. What are the coordinates of point C? Key stage 2 tests (2023) Progress to plotting locations on graphs with four quadrants.
Direction <i>Which way? Moving around, e.g. up/down, forwards/backwards, left, right.</i>	Extend the language of direction, <i>north, south, east, west</i>. Give and follow more complex sequences of directions, using greater accuracy to describe turns.	Computing Write a sequence of instructions, to create shapes and patterns, e.g. using Scratch (https://scratch.mit.edu/). Compass directions <i>The post office is in the direction of NNE. Can you point in that direction? Can you tell someone which direction to walk in to get to the park?</i> Giving directions Successfully give a sequence of directions. Provide alternative routes. <i>Describe how to get to the shop using two different routes.</i>

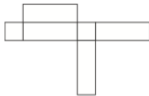
Dimensions Distance, e.g. <i>How far away?</i> Area, e.g. <i>near, in the middle.</i>	Use more precise units, including decimals. Understanding 2D representations of large-scale space, including heights and slopes.	Understanding maps Interpret map contours when planning a route. <i>Which way is less steep?</i> Length and area Organise a walk around the school, using a trundle wheel to measure one kilometre. <i>How many metres in a kilometre?</i> <i>How far is 100 metres: from here to ...?</i> <i>What would be 1km from here?</i> <i>How far do you think it is to the park?</i> Sketches and Visualisation Set an engineering challenge such as the Marshmallow Challenge – use marshmallows and uncooked spaghetti to build the tallest building. Before children start building, encourage them to visualise and sketch their solution (https://www.marshmallowchallenge.com/; Pair et al., 2016; based on Flick & Lakin, 2024).
Transformations rotation (turning), translation (sliding) or reflection (flipping), e.g. a shape or jigsaw puzzle piece to fit or match.	Predict the result of transformations, e.g. how more complex shapes (2D and 3D) will appear when rotated, the result of cutting and folding. Reflect a pattern or arrangement over a diagonal line or in four quadrants.	Nets <i>How many different nets could make a cube?</i> Encourage children to draw using squared paper, then check by cutting and folding. <i>Which of these fold up to make a square-based pyramid. Why/not?</i> 

Which of these nets fold up to make a cuboid?

12 This is a drawing of a cuboid.



Tick the nets that could make the cuboid.

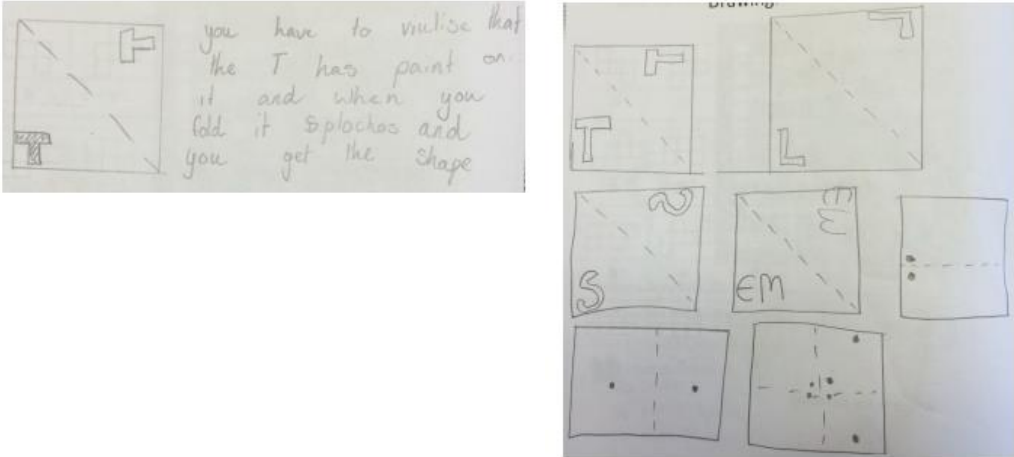


Key stage 2 tests (2023)

Mental rotation
The shape below was turned three quarters of a full turn and ended up looking like this.
What did it look like when it started?

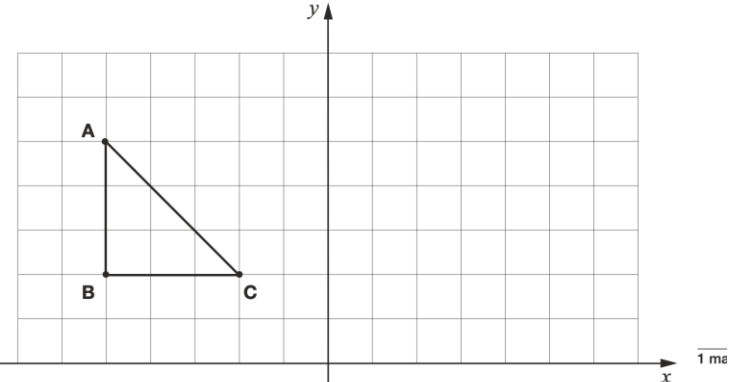


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irection_and_movement_reasoning.pdf](https://www.ncetm.org.uk/media/fczmsmxc/9_progression_map_geometry_position_direction_and_movement_reasoning.pdf)

		Reflection <i>Draw a square with a letter in the bottom left corner, with a diagonal line of symmetry. Draw the reflected letter.</i>  Lowrie & Logan (2023) Folding and cutting paper Predict how to fold and cut a square to make different six-pronged snowflakes. (You need to fold in half then into three, to produce a 60 ° angle.)  https://onelittleproject.com/how-to-make-paper-snowflakes/
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Draw reflected shapes using squared paper.

Amina draws triangle **ABC** on a grid as shown.

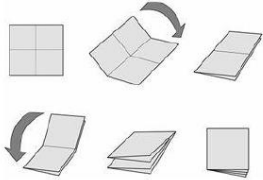


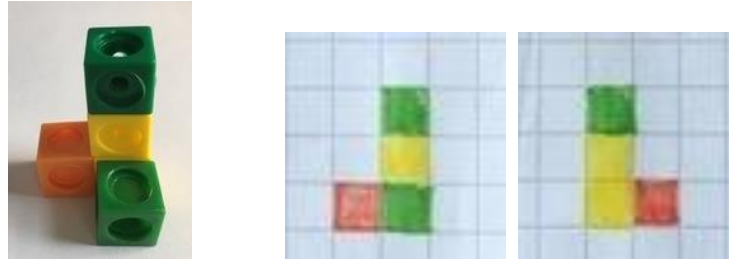
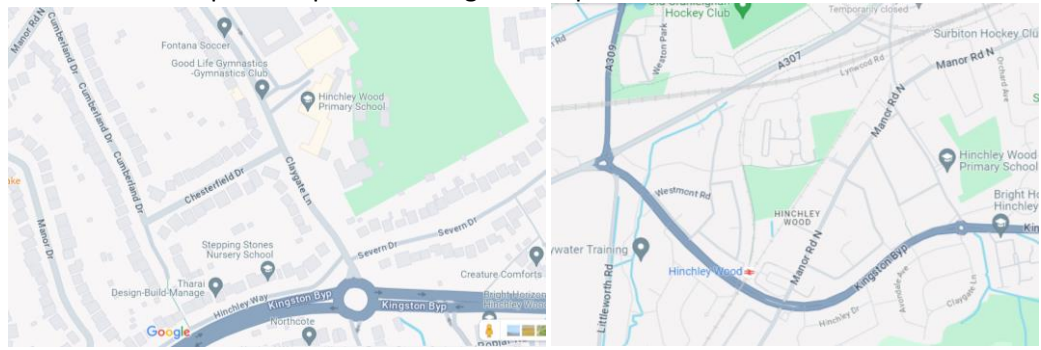
She then reflects the triangle in the y -axis.

Draw the reflected triangle on the grid.

Key stage 2 tests (2023)

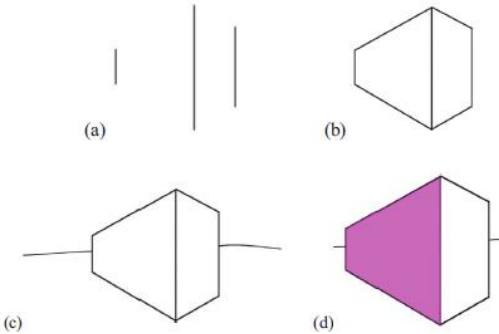
Invite children to design pop-up cards and peer review them before they make them, then afterwards discuss how well their predictions matched their results and why.

		Show children a folded piece of paper with holes punched in. Challenge children to visualise and predict where the holes were punched. Provide paper and a hole punch for children to check their answer. James makes a pattern by first folding a square paper into 4 parts as shown below.  He then punched two holes (one big and one small) through all of the layers. The pattern that he obtained is shown below.  Where did James punch the two holes?  Lowrie and Logan (2023)
Perspective-taking – appearance from different viewpoints Visibility - what can be seen, e.g. hidden or partially visible. Position - <i>where</i> objects are in relation to each other, e.g.	Visualise, draw, describe and interpret different viewpoints, re visibility, position, orientation, appearance, scaling.	Graphs and charts Compare graphs with different scales on the y-axis: e.g. y-axis where the scale is in units of 1 compared to y-axis where scale is in units of 5. <i>What do you notice?</i>

things behind each other appear to overlap. Orientation - <i>which way up?</i> e.g. <i>Upside down, back to front, tipped over.</i> Appearance – e.g. <i>how circles can look like ovals from certain viewpoints</i> Scaling - zooming in and out, e.g. small-world play (toy farms), reading maps.	Visualise and draw a model from different viewpoints. <i>Here are two views of a 3D model (front and back view).</i> <i>Can you draw the left and right views?</i>  based on Mulligan et al. (2020) Design a planted area and draw a diagram/map using a grid. Explore a range of perspectives which might impact what is grown: e.g. map of the sun exposure (https://getbusygardening.com/how-to-determine-sun-exposure/) and after planting, check predictions by measuring hours of sun exposure to each cell of the grid. Scaling Locate the same place or position using two maps with different scales.  (https://www.google.co.uk/maps)
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Perspective drawings

Present children with the three lines in figure a below. *What do you see? Which line is longest? Next, connect the line segments at the top and bottom (figure b below). What do you see now? Next, draw a horizontal line on each side (figure c). What do you see now? Once children recognise it as a building: How many walls does it have? Why can you only see two walls? Referring to figure d: What shape is the pink wall? How do you know it is a rectangle? Which line is longest now?*



based on McGarvey et al. (2024; adapted from Salisbury, 1987)

Ask children to add details. *Add a door, three windows and two trees the same height as the building. How do you make these details look realistic?*

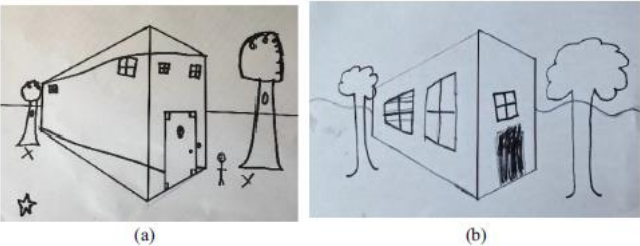


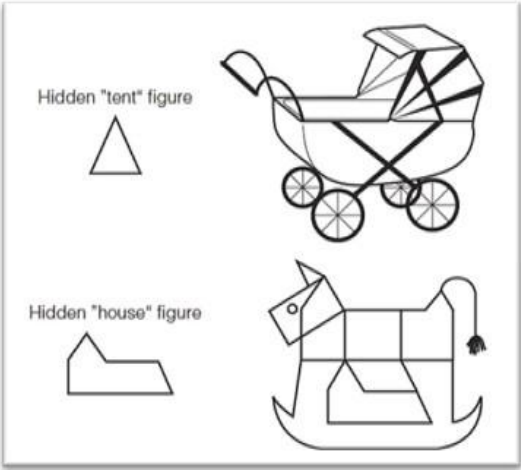
Fig. 9. William's (a) and Noah's (b) structured perspective drawings.

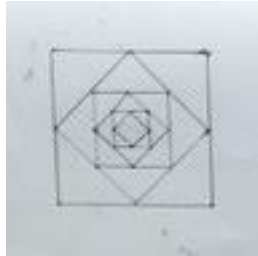
based on McGarvey et al. (2024; adapted from Salisbury, 1987)

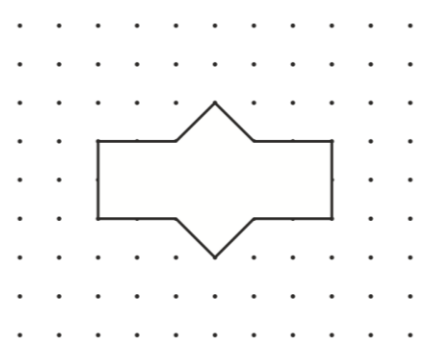
Navigation – e.g. wayfinding, map-making and routes.	Orienteer using maps of unfamiliar environments, rotating to follow directions. Begin to plan and evaluate alternative routes. Being to make use of distant landmarks as well as nearby landmarks.	Route finding When walking a known route, ask children to point in the direction of locations and landmarks that cannot be seen from where they are standing (based on Uttal, 2000). Ask children to learn and describe a route to somewhere and back again (based on Lingwood et al., 2018). Challenge children to find and evaluate alternative routes. Using landmarks Put flags around the school and ask children to place stickers on a map of the school to indicate the locations of the flags (based on Liben, 2009). Challenge children to describe routes using landmarks in the distance as well as landmarks on the route itself (based on Cornell, 2001).
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Shape (objects and properties)		
Lower KS2 (7 to 9 years)		
	Children are learning to	Activities to support learning
Identifying shapes– <i>What?</i> 2D and 3D objects (regular and irregular) such as cups, clothes, jigsaw pieces, leaves and clouds, e.g. <i>circle, rectangle, triangle, heart-shaped; cuboid, cone, ball, roof-shaped.</i>	Define and classify geometric and non-geometric 2D and 3D shapes (including regular and irregular shapes and everyday objects e.g. platonic solids and prisms, clouds and leaves).	Classifying collections Present a collection of shapes, ‘non-shapes’ and ‘nearly-but-not-quite shapes’, e.g. for triangles include cheesy biscuits and a musical triangle. Discuss why the last two examples are ‘nearly-but-not-quite’ triangles, e.g. curved corners, not a closed shape. Shape hunt Spot examples of shapes and ‘nearly-but-not-quite’ shapes in the environment, e.g. a coat hanger is nearly a triangle, the picture on the wall is almost a square. Geostrips and geoboards Explore the range of examples which meet the criteria for a definition of a shape. <i>If a hexagon is a closed shape with six straight sides how many different hexagons can you make? Which one is the regular hexagon?</i> Mathematical and non-mathematical vocabulary Use analogies, e.g. describing a triangular prism as a Toblerone. Minimal definitions What are the necessary but sufficient criteria needed to define a shape? E.g. a rectangle is a 2D closed, straight-sided shape with four right-angles.

Properties including: Size, length, area, capacity and volume e.g. big, tall, wide; Will it fit in? Sides, faces, edges, lines, radius, e.g. straight/ curved, wiggly, zig-zag; equal and unequal. Corners, angles, e.g. points, vertices, right angle, square corner, sharp, obtuse. Symmetry - in 2D and 3D, reflective and rotational.	Discriminate shapes by properties including angle, perimeter, area and topological properties.	Partially hidden shapes and silhouettes <i>What is it? What isn't it? What could it be?</i> Encourage reasoning about properties. <i>I know it can't be a circle because ...</i> Investigate the different silhouettes of a 3D shape from different angles, e.g. a cylinder. <i>What could it be? What couldn't it be? How do you know?</i> Angles and turns Use geostrips of different lengths to make the same angle, to help children understand that the angle is the same even if the lengths are different. Symmetry Identify examples of symmetry in living things, e.g. plants and insects.
Composing and Decomposing Composing - fitting together 2D and 3D shapes, using interrelationships between properties, angles, e.g. with jigsaw puzzle pieces, pattern blocks, nesting containers and construction. Decomposing – disembedding (parts within wholes), cutting and folding, cross-sections, 2D	Create more complex models, constructions and arrangements Visualise and predict folds, nets and cross –sections of increasing complexity, checking using physical objects. Find shapes within complex arrangements, then combinations of lines within overlapping shapes (disembedding).	Construction Cut a square into 4 triangles. <i>How many different shapes can you make with them?</i> <i>Now can you put them back into a square?</i> Make complex structures, e.g. towers with multiple levels including stairs and ceilings (based on Clements & Sarama, 2020). Build a structure using blocks or LEGO® bricks, then take a picture of it and challenge friends to build the structure from the picture (based on Flick and Lakin, 2024). Nets <i>How many different nets can you make for a box?</i> <i>We are going to take apart these boxes; visualise and draw what the nets might look like. Compare your drawing with the deconstructed box.</i>

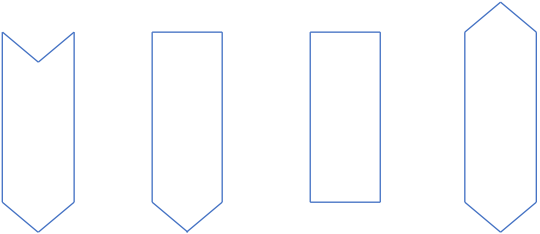
and 3D, e.g. unfolding boxes to make nets and recomposing, <i>which net would make a cube?</i>		Draw a shape Mark points and join them up with straight lines to construct 2D shapes. Draw a regular hexagon by translating the ruler to draw each pair of parallel sides, rather than moving the ruler around. Design a container for an object, draw the net and make the container. <i>Does the object fit?</i> Disembedding Find smaller shapes within a larger image  Karp & Konstadt (1963) Cross-sections <i>What shape will the cross-section be of a cylinder? What shape will it be if you cut it another way?</i>
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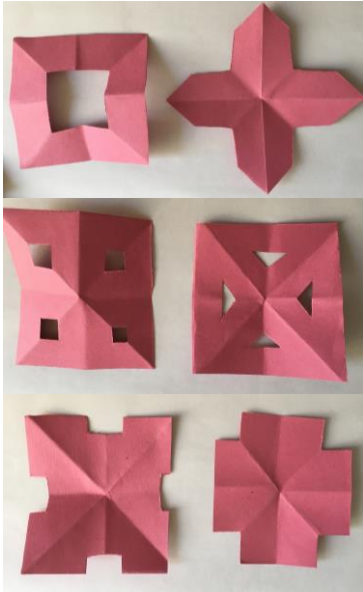
		Cutting and folding <i>Imagine a square: cut the same little piece off each corner. What shape is it now? Go on cutting- what happens? Try the same with a triangle.</i> <i>Imagine a cube: cut a little piece off each corner. What shape is it now? What shape are the faces? How many are there?</i> <i>Imagine a sheet of paper: fold it in half and in half again. Cut off the corner. What shape will you get when you open it up? Which corner did you cut off? (Based on Fielker, 1993).</i>
Scaling Identifying the same item in different sizes, similarity, proportions, enlarging and shrinking, doubling.	Identify the same item in different sizes, (similarity), considering proportions, enlarging and shrinking, doubling and halving. Determine the same location on two maps at different scales.	Number lines Estimate the position of the same number on number lines with different scales e.g. <i>Where is 300 on a 0 to 1000 versus a 0 to 10000 number line?</i> Midpoints <i>Imagine a square: mark the midpoints and join them up to make a smaller square. Keep doing this. What happens? Do the same with a triangle or a pentagon.</i>  Based on Fielker (1993) Doubling and halving <i>Make a 'skeleton' that is half your size out of strips of paper.</i> <i>Find a teddy that is about double the size of this teddy. Is it double the height? Is it double the width? Is it double the mass?</i>

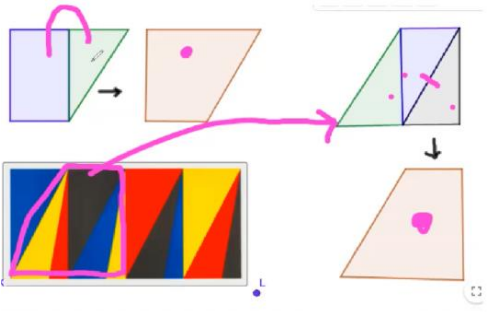
Shape (objects and properties)		
Upper KS2 (9 to 11 years)		
	Children are learning to	Activities to support learning
Identifying shapes <i>What?</i> 2D and 3D objects (regular and irregular) such as cups, clothes, jigsaw pieces, leaves and clouds, e.g. <i>circle, rectangle, triangle, heart-shaped; cuboid, cone, ball, roof-shaped.</i>	Define, classify, draw and visualise geometric 2D and 3D shapes (e.g. using isometric paper).	Classifying shapes Join 4 geostrips together. These could be all different lengths, all the same, two the same, etc.. <i>How many different quadrilaterals can you make? How could you sort them, e.g. kites, rectangles, parallelograms?</i> Isometric paper Draw 3D multilink models on isometric paper.
Properties Size, length, area, capacity and volume, e.g. big, tall, wide; Will it fit in? Sides, faces, edges, lines, radius, e.g. straight/ curved, wiggly, zig-zag; equal and unequal. Corners, angles, e.g. points, vertices, right angle, square corner, sharp, obtuse. Symmetry - in 2D and 3D, reflective and rotational.	Discriminate shape properties, including symmetry and angle measurement.	Symmetry Draw lines of symmetry on irregular shapes. Draw all the lines of symmetry on this shape. Use a ruler.  Key stage 2 tests (2023)

		Shape properties <i>Which shapes have diagonals that cross at right angles?</i> Look at tree and leaf shapes. <i>How could you describe these to help identify them?</i> Discuss topological properties: e.g. the difference between a pretzel and a bagel is topological. <i>Is it always, sometimes or never true, that in a polyhedron, the number of vertices plus the number of faces equals the number of edges?</i> https://www.ncetm.org.uk/media/ettef0hw/8_progression_map_geometry_properties_of_shapes_reasoningv2.pdf Angles and turns Join any 4 geostrips together. Investigate what happens to the area and perimeter when you alter the angles. <i>What is the angle between the hands of a clock at four o'clock?</i> <i>At what other times is the angle between the hands the same? Convince me.</i> https://www.ncetm.org.uk/media/ettef0hw/8_progression_map_geometry_properties_of_shapes_reasoningv2.pdf
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		Use angles in a pie chart to interpret data. 1,200 pupils were asked this question: <i>How important is it to have a break when using a screen?</i> This chart shows the results. <table><tr><th>Importance</th><th>Percentage</th></tr><tr><td>Very important</td><td>41%</td></tr><tr><td>Quite important</td><td>41%</td></tr><tr><td>Not important</td><td>12%</td></tr></table> Key stage 2 tests (2023)	Importance	Percentage	Very important	41%	Quite important	41%	Not important	12%
Importance	Percentage									
Very important	41%									
Quite important	41%									
Not important	12%									

Composing and Decomposing Composing - fitting together 2D and 3D shapes, using interrelationships between properties, angles, e.g. with jigsaw puzzle pieces, pattern blocks, nesting containers and construction. Decomposing – disembedding (parts within wholes), cutting and folding, cross-sections, 2D and 3D, e.g. unfolding boxes to make nets and recomposing, <i>which net would make a cube?</i>	Create more complex models, constructions and arrangements Continue to visualise and predict folds, nets and cross-sections of increasing complexity, reducing support from using physical objects.	Cross-sections The cheese is cut vertically as shown below.  Which one of the following figures represents the shape of the cut edge? (based on Ramful et al., 2017)  For a harder challenge, ask children to predict and sketch the outline (based on Flick and Lakin, 2024).
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	Folding and cutting <i>Fold a paper square in half diagonally, then in half again and in half again. Make a cut as shown.</i>  <i>Which of the shapes below will result?</i>  Choose one of the shapes and predict how to cut the folded paper to produce it (based on Fujii & Majima, 2020).
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		Disembedding Invite children to identify shapes in abstract works of art. These examples are from Max Bill's "Untitled, 1973". <i>Have a look at this artwork. What shapes can you see? For example, can you see rectangles, triangles, parallelograms, squares?</i>  <i>Can you see a trapezium?</i>  <i>Can you see a different trapezium?</i> (from Kus & Newcombe, 2024)
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Scaling Identifying the same item in different sizes, similarity, proportions, enlarging and shrinking, doubling.	Make scale drawings and models.	Doubling and halving <i>Can you make a box that is half the size of this one? How did you do it? Did you halve all the sides, or...? How many of yours fit into the big one?</i> Enlarging and shrinking <i>Find the ratio between the length of your foot and your height. If this is the foot of a giant, with the same proportions as you, how tall is the giant?</i> Length: 32cm Image: Fred Seibert <i>Use strips of paper to make a skeleton that is a quarter of your size or a tenth of your size.</i> Draw an enlarged image of an object, e.g. flower, snail shell. Draw the object on centimetre squared paper and then copy cell by cell onto a larger grid. Scale models Invite children to construct with blocks, or draw, a small version of a building to match a picture or a photograph, taking care that the features are in similar proportions.
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