Spatial Reasoning in early childhood

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EARLY CHILDHOOD MATHS GROUP

The development of spatial reasoning

Spatial reasoning involves our interpretation of how things, including ourselves, relate to each another and our spatial environment. We engage spatially all the time, from babies reaching for a toy to adults calculating how much paper to cut to wrap a present. We use it every day to navigate in and around our environment, to identify, manipulate and manoeuvre objects as well as to communicate and make sense of visual images and schematic maps and diagrams. The term 'spatial reasoning' is often used interchangeably with 'spatial thinking' to include spatial awareness and spatial representation. Spatial reasoning appears in many curricula within 'Shape and space' or 'Geometry' and may be defined as: *the ability to recognize and (mentally) manipulate the spatial properties of objects and the spatial relations among objects* (Bruce et al., 2017:147).

Spatial reasoning involves interpreting images and creating representations, enabling us to predict and solve problems. It includes recognising objects by their shape, finding things, navigating around, fitting things together and into spaces. This, later, also involves interpreting photographs, diagrams and maps. Spatial reasoning involves both static (still) and dynamic (moving) relations within and between objects or features: for instance, knowing what an apple might look like when cut in half in different ways and knowing that the right turn will be on your left on the way back.

Spatial reasoning is important. It develops early, beginning with babies' awareness of space and distinctions between shapes, the development of concepts such as *round* and *pointy*, *near* and *far* and the ability to visualise objects and locations. There is overwhelming evidence of a link between spatial reasoning and achievement in mathematics, science and technology as well as the arts (Wai, Lubinski & Benbow, 2009). Spatial reasoning can be taught to all children (Uttal et al., 2013). However, girls (Newcombe, 2020) and children from 'low-income homes' (Verdine, 2014; 2017) are 'harmed in their progression in mathematics' by having fewer opportunities to develop spatial reasoning (Sarama & Clements, 2009: 161). Verdine et al. (2017:102) suggest the development of spatial reasoning is especially important in the early years, when significant skills are developing and that 'optimizing spatial performance may be an underutilized route to improving mathematics achievement'. It is therefore important for practitioners to understand the importance of spatial reasoning and to know how to support children's development of spatial reasoning.

The ECMG has reviewed recent international research into the development of spatial reasoning from birth to seven years of age and has developed this into a trajectory of early learning experiences. Contexts for these include outdoor play, construction and puzzles, which are well-established in early years practice and will come as no surprise to practitioners. This guidance aims to clarify the mathematics in early spatial experiences such as these, suggesting what to emphasise in order to support the development of children's spatial reasoning in sensitive, appropriate and playful ways.

For many children, spatial thinking develops from recognition to visualisation and representation:

- **Recognition** of spatial and shape properties through sensory experiences
- **Visualisation** - imagining and manipulating spatial information in the mind's eye**,** involving memory and prediction
- **Representation -** gesture, language, modeling and 2D representations including pictures, drawings, maps, graphs and schematic diagrams.

Key aspects of spatial reasoning include:

- **Spatial relations:** position, direction and routes, perspective-taking, transformations
- **Objects and images:** composing and decomposing shapes, transformations (including symmetry and tessellation)

In practice, these overlap. Patterns and measures often involve spatial reasoning. **Patterning** is often experienced spatially by young children when they recognise patterns in what they see and when arranging patterns with objects. The spatial reasoning in **measures** involves the size and distance. For older children distance, length and area involve proportion, e.g. *in the middle, a third of the way along,* and identifying shapes which are *similar*, having the same proportions but in different sizes (spatial scale).

Spatial Reasoning involves:

Spatial relations

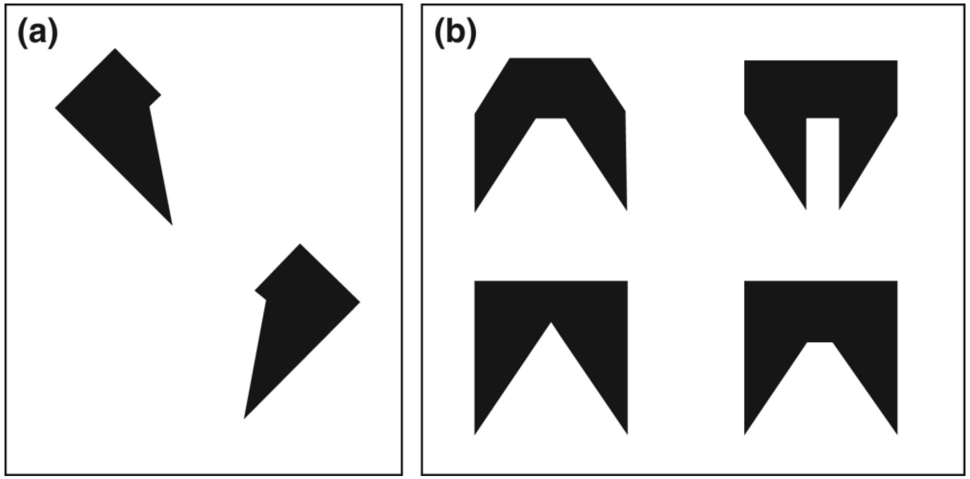
- **language of position – *Where?*** in relation to one or two things e.g. *next to, between*; relative to the viewer, e.g. *in front of, behind*
- **distance – *How far away?*** Length and area, e.g. *near, in the middle*
- **direction – *Which way?*** Moving around, e.g. *up/down*, *forwards/backwards, left, right*
- **changed orientation** - ***Which way up (or round)?*** *Upside down, back to front, tipped over, this way up*
- **composing** - fitting together 2D and 3D shapes, using interrelationships between properties e.g. with jigsaw puzzle pieces, pattern blocks, nesting containers and construction.
- **movement and rotation -** e.g. turning, sliding or flipping a shape or jigsaw puzzle piece to fit or match
- **symmetry recognition -** in 2D and 3D, reflecting, pattern making, block-building
- **perspective-taking** – appearance from different viewpoints
    - visibility (*what* can be seen, e.g. hidden or partially visible)
    - size and distance (*how* things far away look smaller)
    - position (*where* objects are in relation to each other, e.g. things behind each other appear to overlap)
    - appearance (e.g. *how* circles can look like ovals from certain viewpoints)
- **scaling** - zooming in and out, e.g. small-world play and map-making
- **navigation –** e.g. way finding and routes

Objects and images

- **identifying– *What?*** 2D and 3D including the shape of everyday objects such as cups, clothes, jigsaw pieces, leaves and clouds, eg. *circle, rectangle, triangle, heart-shaped; cuboid, cone, ball, roof-shaped.*
- **properties** including:
    - size, e.g. *big, tall, wide*
    - sides, faces, edges, lines; *e.g. straight/ curved, wiggly, zig-zag*
    - corners and angles e.g. *points*, *vertices, right angle, square corner, sharp*
- **cutting and decomposing shapes –** to make new shapes, parts within wholes, bending and folding (e.g. making cylinders with paper strips, unfolding boxes to make nets and then refolding, halving shapes, creating symmetries)
- **structure -** symmetry, cross-sections, 2D to 3D
- **scaling –** identifying the same item in different sizes, enlarging and shrinking

The importance of spatial reasoning in learning mathematics

While the link between spatial reasoning and mathematical learning in general is well established by research, the nature of that link is not clear, particularly for number (Hawes and Ansari, 2020). There is evidence that we use the same area of the brain to visualize and to represent numerical relationships. Since many concrete representations of number are spatial, such as manipulatives, number lines and graphs, visualization skills also help in working with these. Research has found that five year olds' ability to mentally rotate and combine shapes predicted their accuracy in putting numbers on an empty number line when they were six (Gunderson et al, 2012). In this example, you need to mentally manipulate the two shapes in (a) to find which shape they combine to make from the four options in (b).



Children's Mental Transformation Task (Figure from Levine et al. 2016)

When deciding where to position numbers on a line, it also helps to identify 'halfway' points, requiring proportional thinking. Number lines can be on different scales, to include fractions or extending to larger or negative numbers, so it is useful if children can mentally enlarge or shrink images, or 'zoom in' and 'zoom out'. It seems that spatial skills, including fractioning and scaling, are likely to help in interpreting a range of mathematical diagrams and graphs.

Visualization skills also help people to create schematic diagrams to represent all kinds of relationships: for instance the relationship between members of a family are shown by a tree diagram or the relationship between observation, assessment and planning may be shown as a circle with arrows. Creating such diagrams can help to solve unfamiliar problems: interestingly, children who are good at visualising draw effective diagrams with less pictorial detail than those who are not so good at visualizing (Hegarty & Kozhevnikov, 1999). For instance, 12 year olds were given the problem:

> *A balloon first rose 200 meters from the ground, then moved 100 meters to the east, then dropped 100 meters. It then traveled 50 meters to the east, and finally dropped straight to the ground. How far was the balloon from its original starting place?"*

Those who were better at mentally rotating and transforming shapes drew more abstract diagrams than others, who might draw a picture of a hot air balloon, which did not help to solve the problem, as in the examples below.

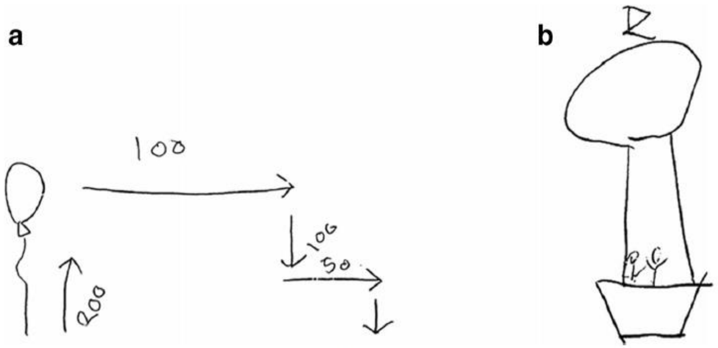


Fig. 5 An example of a visual-schematic representation (**A**) vs. a pictorial representation (**B**)

Figure from Hawes & Ansari (2020)

It may therefore be that spatial thinking is useful in problem solving generally, and not just in mathematics. It also seems that spatial skills are more engaged when people are solving unfamiliar problems learning new concepts, which they later understand more abstractly (perhaps with words or symbols). For instance, ideas of multiplication and ratio may first be envisaged spatially, then understood more symbolically. Visualisation may therefore play an important role in the general learning process, rather than being useful for particular areas of mathematics (Mix, 2019).

Supporting children's spatial reasoning

Many aspects of spatial reasoning are embedded in children's everyday lives as well as in early childhood practice. Recognising children's spatial competencies and interests allows us to build on their strengths, supplementing and enhancing spatial reasoning opportunities within a broad range of early experiences. Whilst activities can offer rich opportunities for spatial reasoning, it is *how* children and adults engage with these that fosters development. From birth, children are building up knowledge through embodied experience of shape and space. For example, children may be interested in boxes, or in posting items into drawers; they may be exploring ideas of inside or what fits. The adult role is crucial in following the interests of children, recognising and sometimes drawing children's attention to spatial elements within their play and everyday activities. This is a complex and nuanced role where adults might spontaneously begin or join in with children's spatial exploration or use spatial words and gestures in context to encourage children to engage in spatial reasoning.

Children's bodily awareness and physical experiences underpin the development of their spatial reasoning. The large-scale movements that are crucial for physical development, and commonly encouraged during outdoor play, have been found to be important for very young children to learn to interpret views from different perspectives and to visualise these (Oudgenoeg-Paz et al., 2015). Some children may have a strong drive to repeat their actions over and over again, such as moving or throwing things. Athey (1990:37) describes these as schemas: 'a pattern of repeatable behaviour into which experiences are assimilated and that are gradually co-ordinated'. These occur at different times for different children: for example, some children may show a preference for lining things up or a fascination for putting objects inside other things. As children grow, they draw upon a range of tools to assist them with their spatial thinking and make it more efficient (including words, gestures, images and symbols). Language and gesture are particularly helpful in forming concepts about shape and space. Some commonly over-looked aspects in early mathematics curricula are perspective-taking, symmetry, scaling, and navigation and these are discussed in more detail below.



Physical development

Babies and young children use movement and senses to explore their worlds and communicate their thinking. Spatial awareness underpins spatial reasoning as an embodied process; feeling 'my body in the world' so that I can act upon it. Spatial reasoning cannot develop without strong body awareness and strong awareness of the environment. This awareness grows through the integration of sensory systems, providing the body with a combination of internal and external information in order to visualise or mentally represent the environment. It takes a long time to develop and automate these processes and the mental representations children produce need to be updated constantly as their bodies grow and change. This is why young children need such a lot of time to be physically active, gaining feedback from the world around them and experiencing the world's response to what they do.
Spatial reasoning is underpinned by the intricate linkage between the internal and external information provided by three sensory systems:

- The vestibular system – a motor sensor system that registers movement of your body in the world. It is critical for understanding how your body moves in space and how it understands space including balance and awareness of being upright.
- Proprioception – body awareness. It provides an embodied understanding of the location of parts of the body and the body in space.
- The visual system - supporting static and dynamic understandings of shape and space, e.g. sensing distances between objects. Hearing is also critical for getting feedback from the environment. Children with visual or hearing impairments will rely more on other senses.

It is crucial that adults recognise the importance of the development of these senses in babies and young children and demonstrate this by providing plenty of space, time and opportunity for children to be physically active throughout the day. 'Embodied learning' means that children physically encounter and experience phenomena such as 'round', 'bumpy', 'upside-down' or 'inside' and build up their understanding of what these terms are through their senses before, and where appropriate, at the same time as adults provide the vocabulary to support a concept.

Gesture and language

The development of children's spatial awareness and reasoning is enhanced by the use of language and gesture. More precise language helps children to focus on shape properties and spatial positions and to conceptualise these. Research shows that this process is dependent on the quality of children's experiences, the adult's role in providing appropriate language and also on the use of gesture by both children and adults. This starts with adults paying attention and responding to babies' gestures and eye gaze, e.g. for instance when a baby waves a hand to show that they want to climb in a box and the adult acknowledges the child's gesture, *'You want to go **in** the box*?' Toddlers with more experience of large-scale movement and toys can understand more sophisticated terms, such as *between* which relates a position to two other objects, or *in front of* and *behind*, which are relative to the viewer (Oudgenoeg-Paz et al., 2015).

Gestures provide initial ways of communicating spatial ideas: these are often requests (or commands!), showing what babies would like to happen (e.g. pointing to where they want to go or putting their arms up if they want to be carried). Adults can supply spatial words for these, like *in* and *up*. Gestures are important ways of supporting language, for instance pointing to an object's location, moving a hand around when saying *curved,* or turning bodily to explain *left* or *right*. Many studies have found that gestures help both adults and children to describe and understand features or movements of shapes (e.g. Bower et al., 2020). Adults can encourage children to use gestures, such as putting hands close together for *small* and far apart for *big*: including physical activity in this way enhances children's learning (Levine et al., 2018).

https://depositphotos.com/categorycities.html

There are three main types of gestures relevant here:

- gesturing an action required or visualised (e.g. rotating a shape to fit in a puzzle).
- tracing the outline of shapes (as used by Young et al., 2014), highlighting properties and linking these to spatial words and concepts.
- supporting language, helping to communicate otherwise difficult spatial concepts and so providing a bridge to learning new concepts or words (Singer & Goldin-Meadow, 2005).

on the bridge

in front of the bridge

behind the bridge

Spatial language, supplemented and supported by gesture, can be introduced in everyday and play contexts: for instance getting dressed may involve spatial ideas of *back to front* and *inside-out*. What is developmentally appropriate for individual children will vary according to their experiences. Most children understand *in, on* and *under* at three years old, but only come to understand *between* and *behind* when they are four, unless they have had a lot of large-scale movement experience (as mentioned above). Some children will be seven years old before they can use relative terms like *left* and *right*, but many four year olds will understand these when accompanied by gestures. Learning spatial vocabulary helps children to conceptualise the distinctions between different positions and properties (Farran & Atkinson, 2016). This suggests that when young children experience difficulties, for instance in copying a model with pieces at right angles to each other, we might supply a term like *across*, and also a gesture like crossed hands, to help them to conceptualise this relationship. It seems that language helps children to hold more things in mind, so that, for instance, they can remember the shape properties while focusing on moving them to fit them together (Pruden et al, 2011).

Models to copy in order of difficulty (after Verdine et al., 2017)

Spatial language can:

- help children to use and recall spatial information, such as relationships between objects e.g. *next to, in front of* (Feist & Gentner, 2007);
- improve children's conceptual understanding by refining spatial categories, e.g. the difference between *on* and *in* (Farran & Atkinson, 2016);
- help to draw children's attention to the relevant spatial attributes when problem solving, e.g. relative positions of blocks when copying constructions (Bower et al., 2020);
- highlight the spatial relations that underlie mathematical concepts (e.g. numbers on a number line; Mix & Cheng, 2012).

Exploring and communicating about a range of shapes involves a wide variety of language beyond standard geometrical terms, in order to identify them and tell them apart, especially if we are describing everyday objects and routes. For example, we use informal language such as describing *a bendy, wiggly, twisty* or *zigzag* path, making a *loop* or a *circuit* and we use analogies like *a dogleg* junction (in New Zealand signs advise traffic lanes to *merge like a zip*). When distinguishing leaves and their growth patterns, we may use analogies such as *hand-shaped, spear- or heart- shaped*, or terms like *smooth-edged*, *serrated, toothy*, *opposite* and *alternate*. When doing jigsaw puzzles, we might refer to the *corner* piece or *straight edges* to differentiate pieces, or use our own terms like *sticky-out bits* and *holes* to describe the piece we are searching for or explain why a piece cannot be the right one for the space. Researchers have found that introducing children to more irregular shapes encourages them to make finer distinctions between shape properties (Verdine et al 2019). This suggests that we should be describing and categorising shape and spatial properties in a broad way, encouraging children's own voice and creative language or analogies, as well as introducing mathematical terms when these make useful distinctions.

Individual and group differences

Individuals will vary in their spatial abilities according to their characteristics, development and experiences, in ways which are not yet very clear, and which vary for different aspects of spatial reasoning. With regard to gender, although there are differences between men and women regarding mental rotation, there are virtually no differences between pre-school boys and girls; a difference emerges significantly in adolescence but we do not know whether this is due to developmental or environmental causes. For adults there are smaller or no gender differences for some aspects of spatial reasoning, with no differences with some visualisation tasks, such as paper folding and less for navigation than might be expected from popular stereotypes: this suggests there is no such thing as a general spatial ability which some have and others lack. However there is evidence of differences in the experiences of boys and girls, with regard to physical activities, parental language and degree of challenge (Newcombe, 2020). Regarding children's social backgrounds, children from poorer backgrounds tend to enter schools with lower spatial abilities and may lack experience with spatial resources, and toys. Middle class parents may use more spatial language, but differences in children's experience of outside spaces will depend on individual situations. However, children from low SES backgrounds tend to make greater progress in response to teaching (Clements and Sarama, 2020; Verdine, 2017). Children with special needs, such as with visual impairments, show similarities and differences to visually-able children, but spatial concepts and mental representations can be built through movement and touch: for instance directing robots helps children to visualise spatially (Sarama and Clements, 2009.)

In general, it is important to develop spatial reasoning with children from birth to seven, because spatial training is optimally effective for young children (Uttal et al., 2013). Spatial teaching is also particularly effective for children of parents with a lower educational background (Schmitt et al., 2018) and for addressing gender differences (Sarama & Clements, 2009). Teaching spatial reasoning in the early years is also particularly effective for children from low SES families (Heckman, 2006), therefore providing an opportunity to reduce the attainment gap between children from the more and less advantaged communities.

Environments for spatial reasoning

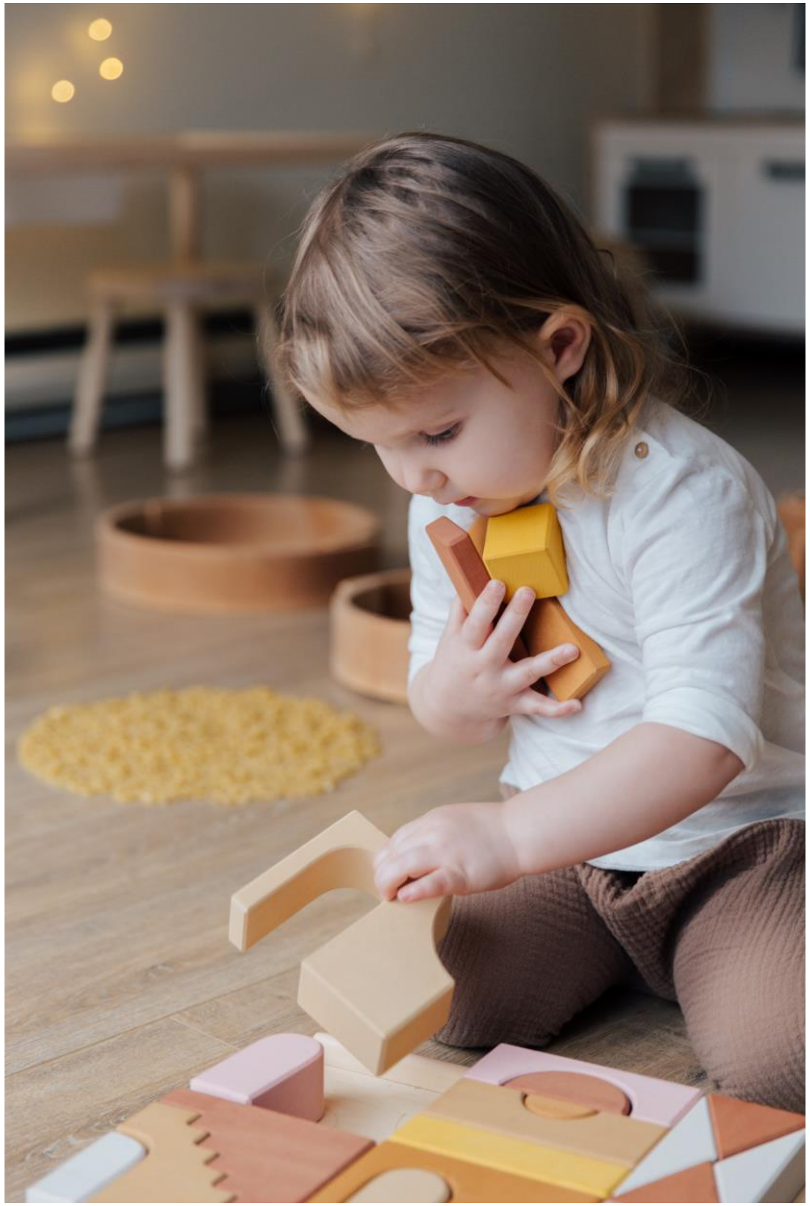
This section considers the:

- **physical environment** indoors and outside in terms of layout and resources
- **emotional environment** that plays a significant role in supporting the development of dispositions necessary for successful mathematical learning
- **language environment** where sensitive and timely adult interactions support the development of spatial concepts and spatial reasoning.

Environments are created and maintained by adults. Those who recognise the importance of spatial reasoning are more likely to plan for and encourage it and those who notice children's spatial play are more likely to give children permission, e.g. to squeeze into spaces or enjoy different perspectives. Adults can positively influence the emotional environment by ensuring that children have ample time to engage in spatial play and by valuing and providing for repetition and revisiting of ideas. Creating an atmosphere of safety and security supports positive attitudes to trial and error and to risk taking. Further, adults need to model curiosity and encourage that disposition in children by showing genuine interest in what children are exploring and finding out.

Some resources to include in provision will be very obvious such as puzzles, block play and shape resources that might be offered in a maths area. What might be less obvious are the opportunities that are, or could be available through setting routines such as lining up or group times and in other areas of provision, e.g. in the book, pretend play, art, workshop or small world provision. Where resources and experiences have the potential to support spatial thinking but it is not a given, it is more often the role of the adult to sensitively interact in children's play, providing the words alongside the experience or drawing attention to an aspect of spatial thinking. Given that girls and children from low socio-economic-status backgrounds are more at risk of missing out on the opportunities it would be expedient to take notice of these cohorts of children.

Through repeated exposure to the physical environment, resources and experiences children develop an embodied understanding of spatial phenomena, e.g. they gain understanding of shape properties such as the roundness of a ball or the curved property of a piece of guttering by



handling these materials. To appreciate the potential for spatial awareness and spatial reasoning within the layout of the physical environment, a starting point might be to consider it from a child's perspective, e.g. considering possible routes through the classroom or outdoor space, views from one area of provision to others, vantage points and enclosed spaces that children choose to explore. The outdoor environment is especially suited to physical play and play with open ended materials that can be transported and therefore offers rich opportunities for children to build mental maps of their surroundings. To encourage this, adults may decide to store materials e.g. for den building or transporting water away from the areas where children play with them. Beyond the setting gates there may be further opportunities to support spatial thinking in terms of routes and journeys. Where these routes are frequently travelled and adults allow children the time to explore features of interest to them on the way, adults may notice children developing rituals throughout the familiar journey, e.g. walking along a specific path edging or jumping on a manhole cover. An essential aspect of an enabling environment for spatial awareness and spatial reasoning is that adults recognise the importance of repetition and exploration such as this.

Simon Gregg

Families

Families support early spatial reasoning development in a range of ways, often without realising this is what they are doing. When family members engage in early physical play and other experiences with children, this supports early spatial exploration and sense-making. Taking children around their locality, rough and tumble play and providing time and space to explore their bodies and the space around them makes a vital contribution to children's early spatial learning.

Families spend time on the floor with babies, join in with their physical play and move babies around so they experience movement and seeing the world from different viewpoints. This continues to develop with hiding games and encouraging children to explore larger spaces through taking children to gardens and parks where they can run, jump and climb as they become more mobile and independent. As children mature, families often also engage in more structured spatial play with children, such as using jigsaws, puzzles and blocks.

Parents and caregivers use spatial talk frequently with young children as they go about their everyday lives and when playing with them (although the amount varies considerably between families). This talk includes a range of spatial terms such as *up* and *down* (direction); *big* and *small* (size); *edges* and *corners* (features and properties); *upside down* (orientation); *turn* and *turn over* (transformation) (Ho et al., 2018). Where parents are provided with support for what spatial activities they could do with their child and what they might talk about, this supports children's spatial talk (Polinsky et al., 2017). This might be as simple as recommending or loaning small world figures and vehicles to be included in shared construction play as this provides contexts to use spatial words as the people or cards are moved *in* and *out, up* and *down* or positioned *next to* or *on* structures within the play (Ferrara et al. 2011). Children whose families use more spatial words with them use more spatial words themselves (Polinsky et al. 2017, Pruden et al. 2011). These words help the child to form a conceptual understanding of the spatial idea which they use in their spatial thinking. For example, the word *inside* is associated with interior spaces, being surrounded or encased and as the opposite of *outside*. This word gives a label to this concept to support clarity in spatial thinking with the word *inside* coming to mind when deciding where to place an item or when wondering where an item might be, for example. In addition to spatial play and everyday experiences, using spatial language when sharing a picture book is a good way for parents to support children's spatial reasoning (Szechter & Lieben 2004).

Of course it is not just those in a parental role that support children's early spatial thinking development. Other children in the family often engage in physical play, model using spatial words and play spatial games and puzzles with children. Shared construction and ball games, for example, are rich opportunities for spatial learning where communication of ideas are necessary within the shared experience and activities can flow, revisited and becoming extended over time. Similarly, members of the extended family enjoy action songs, play games and go on journeys with children. They support children's spatial learning through pointing out and talking about landmarks (places along the way), perspective (how places appear from where they are or will be) position (where something is) and direction (which way they are going).

Practitioners in settings support children's spatial reasoning through their interactions and provision but also through their communication with children's families. This needs to be planned into the regular time to discuss individual children's mathematical learning and not on an ad hoc or opportunity led basis as some children and their families might get missed, particularly if they are shy or less confident with mathematics. Practitioners support spatial learning in partnership with families by:

- valuing children's home spatial learning,
- sharing examples of their child's spatial learning in setting
- providing ideas for extending spatial learning at home

Valuing home learning includes listening to children and their families talk about the physical play and journeys they have experienced together. This may include learning some spatial words in the child's home language and using gesture alongside these and the English words. Practitioners might encourage families to share photographs with them of their child climbing and exploring, for example, or a model they have made at home using construction toys.

Examples of children's spatial learning can be shared with families through encouraging children to engage with tasks that can be shared with their families. This might include providing simple drawn maps and photographs of the setting for new children and their families to take home. One nursery, for example, encouraged children to create maps of the nursery to show their families where their favourite places and things were which the children enthusiastically shared with their families. Supporting children to take their own photographs in the setting and whilst on walks with practitioners is another good way of children sharing their spatial learning with families.

Practitioners can also support home learning of spatial reasoning through providing pictures books (such as those on the 'ECMG spatial reasoning book list') to be borrowed and shared at home. These could have key spatial words glued insider the cover or on a bookmark to help emphasise these to families alongside enjoying the story. Practitioners also might ask families to take photographs of their journey to the setting which can be made into a map or sequenced by the child (and perhaps taken home to share with families afterwards), encouraging children to sequence the landmarks that are important to them along their journey.

Books

Children's books provide meaningful contexts to explore spatial reasoning. Adults and children can enjoy books together, using spatial language and exploring spatial problems (such as looking *under* the bed or *behind* the door in a lift-the-flap book). Some books are particularly helpful for drawing their reader's attention to specific types of spatial reasoning, such as perspective-taking or navigating. Our 'Spatial reasoning book list' makes some suggestions. Acting out stories or ideas from children's books, using props or pictures, can help children to move their bodies to explore the spatial aspects physically for themselves. Making their own maps or plans of the places or story sequence from a book can be an enjoyable activity for children, where they can represent what they found in the book or can think creatively to invent their own places or events using their imagination, perhaps extending the story and creating alternative narratives.

Spatial reasoning across the curriculum

Spatial reasoning supports and is developed through learning in curriculum areas other than mathematics. Our 'spatial reasoning across the curriculum' document suggests ways in which spatial reasoning can be applied and developed in contexts and activities which do not have a mathematical focus. These are presented as the generic ways in which they support learning (near the centre) and then subject-specific ways are arranged in a wider ring, referenced to school subject areas on the inside (National Curriculum in England) and early years areas of learning in the outer ring (EYFS in England). Recognising, incorporating and extending spatial thinking across the curriculum can benefit both children's mathematical understanding and their understanding in other areas too. Spatial reasoning connects to a range of areas of learning so developing it with young children from birth to 7 years is a way of building foundations for learning in subject areas beyond mathematics for later schooling and indeed careers and everyday life.

Aspects of spatial reasoning

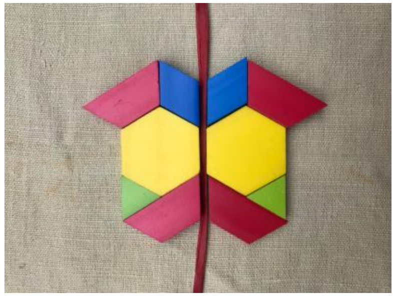
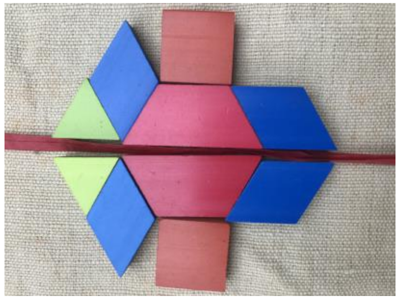
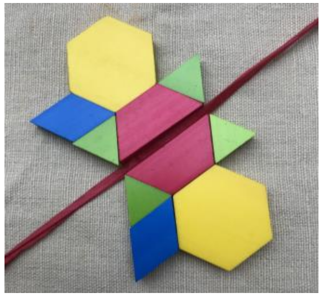
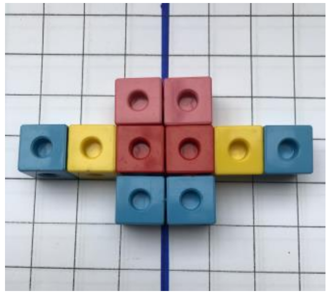
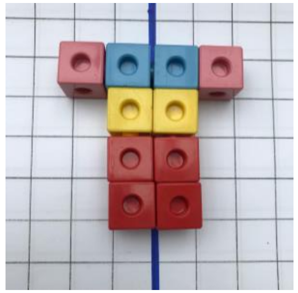
The ECMG spatial reasoning trajectory contains many aspects of spatial reasoning that will be familiar to most practitioners but some may be less familiar. We feel that symmetry, perspective-taking, scaling and navigation are worthy of elaboration.

Developing a sense of symmetry

Symmetry is an area of spatial reasoning which would benefit from greater attention. Bruce et al. note that, '*research in Psychology shows that children come to school with an already strong capacity for identifying symmetry (Bryant, 2008), suggesting the potential for much more in-depth learning in this area.'* (2017:152). They point out that symmetry is of major importance both aesthetically and scientifically, as well as in higher mathematics. Young children often create images with symmetry or build structures in block play which display 'balance'.

Alison Borthwick

Children are fascinated by reflection when they engage with mirror play. Sometimes they create symmetrical arrangements accidentally, as the result of adding objects with two hands or by repeating an arrangement: true reflective symmetry involves a mirror image or the idea of reversal, 'the same but the other way round'. The patterns below (from the work of Moss et al., 2015) show a possible progression in children's understanding of 2D reflective symmetry. Sarama and Clements (2009) suggest that five year olds can often flip shapes to match an image over a vertical axis (but may do this in the wrong direction), whereas many six year olds can reflect images over a horizontal axis and seven year olds over a diagonal axis. Opportunities to explore symmetry on a square grid, as below, can lead to a later understanding of coordinates.

Progression in understanding symmetry can also be found in children's drawing, as shown by the work of Mulligan et al. (2020). They studied the development of four- and five-year-olds understanding of symmetry through asking them to draw a face, then cutting the drawing in half and asking children to draw the other half. They categorised the responses, as shown below, in terms of levels of pattern and structural awareness. At the earlier levels, children drew all or repeated some features on the other half of the face, whereas at the advanced levels they could draw the mirrored image and add to both sides.

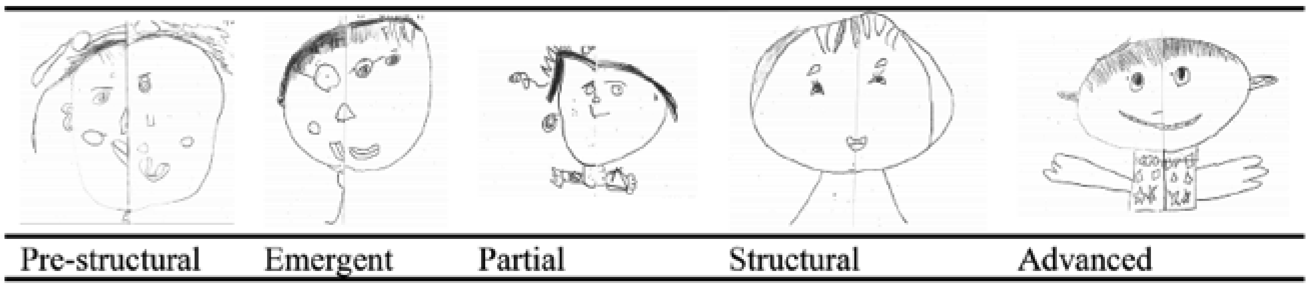


Fig. 7 Student's symmetrical drawings of their face

Mulligan et al. (2020:673)

Alison Borthwick

Children's patterns can also show rotational symmetry, as with the example (left). This may not be intentional and may arise from reflective symmetry or from a radiating schema, but such experiences may support later learning. Research studies about symmetry suggest that young children can develop more sophisticated understanding of symmetry, and may be responsive to greater creative challenges if they are given experience with age–appropriate materials and contexts.

Perspective-taking

Perspective is about how things appear to us from where we are. It involves knowing how things appear differently depending on where they are (position). It also involves whether or how much we can see from where we are (visibility). Perspective-taking requires cognitive flexibility as it involves considering how things look from a specific perspective. Imagining an alternative perspective is more natural than one might think and children are more likely to be able to do this if there is another person or character that the child is considering (as they form a sort of relationship with the character and can try to see things from their perspective or through their eyes).

Peekaboo

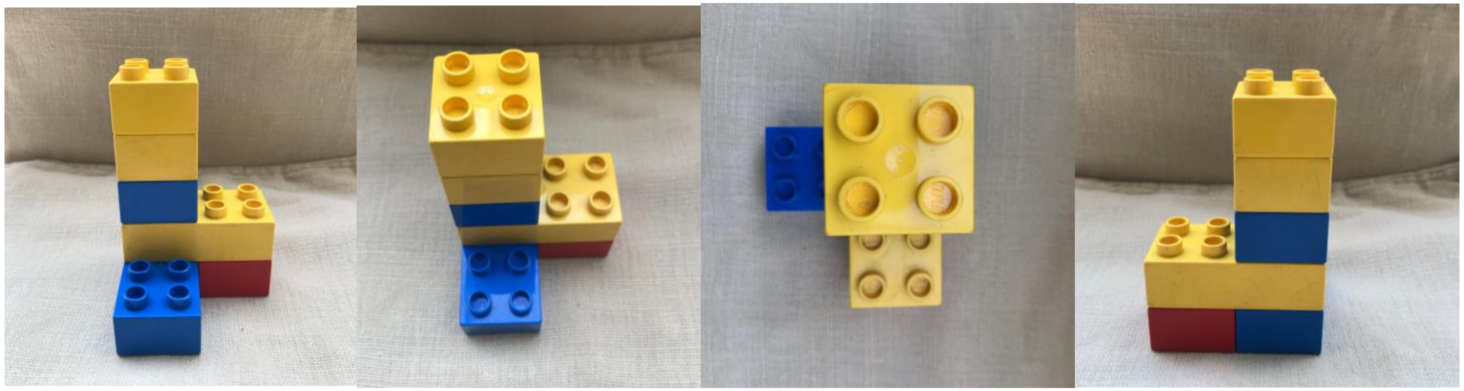
Looking through

Can the girl see the car?

What can the cow see?

Very young children often enjoy viewing their environment from different perspectives, enjoying turning upside down or being lifted up to experience seeing the world from these different vantage points. Children interpret different perspectives as they move around, seeing the world from different locations. To understand an object better, they may want to turn the object or move around it or look underneath or inside to improve their sense of it by viewing it from multiple perspectives. Being able to stand on stones, logs, stools, a low wall or hill provides an interesting change in perspective, as does lying on the ground or crouching down. As children grow older, toys can be used to support children's perspective-taking. A simple game to play with older children is to hide a toy in the environment and tell them what the toy can see and then challenge the child to locate the toy from this (e.g. Spiderman can see a big plant with green leaves in front of a window through which he can see a traffic light far away). Video (including webcams) and photographs can be really useful for encouraging children to engage in perspective-taking in a familiar environment, finding where an image is taken from or what is different between two perspectives.

Young children enjoy games where things and people shift between being hidden and visible (such as peekaboo). This develops as children mature to being able to imagine the part of an object that is hidden or recognising an object even though only part of it is visible. Perspective-taking is linked to orientation so that children can remain 'unconfused by changing orientations' (Fujita et al, 2020:237). For older children (typically from six years), perspective-taking works alongside mental rotation to support them to visualise objects presented in different orientations (Cross et al, 2009). An example is the Lego model below, photographed from four different viewpoints.

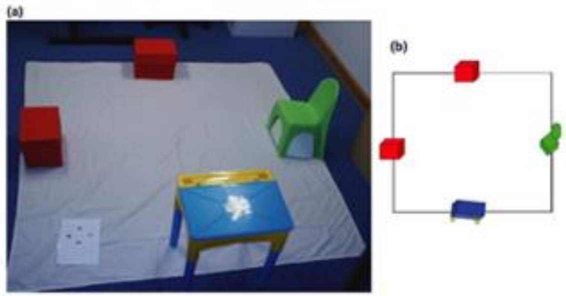


After Boriello (2018)

Scaling

Scaling means working between different size versions of the same thing, for example, toy versions of cars and playfood. This begins with symbolic and small world toys, where children's play shows that they understand that the small version represents that thing in the real world and behaves in the same way. This develops towards an understanding of the relationship between different scaled versions, such as appreciating how large a dinosaur would have been in comparison to the toy model. This extends to understanding the spatial relationships represented by maps or models of real places, including scaled distances and proximity to each other. In these instances, they are usually representations of real places or arrangements, such as shown below where the diagram (b) represents the actual arrangement (a).

Gripton (2020) DOI: 10.1080/0305764X.2020.1745149



Farran et al. (2010). DOI: 10.1111/j.1467-7687.2009.00894.x

Scaling can involve working with relative sizes (scaled objects) or relative distances and positions (sometimes involving perspective-taking) based on a scale model or map. Scale models or maps require 'dual representation' (DeLoache 2000) where the child needs to understand that the model represents the real place (that both are the same but a smaller or larger version).

As younger children (such as toddlers and two-year-olds) are developing their spatial reasoning skills, they tend to try to use miniature toys or objects as though they are full-sized objects (Rosengren et al 2010), however the more experience they have with small-scale objects, the more their sense of scaling develops (Pritulsky et al, 2020), highlighting the importance of small world play for young children. Young children recognise scale models of animals, vehicles and dolls' houses and can often choose the correct chair, bed or bowl for each of the three bears, based upon its size and their knowledge of the story context. They can also interpret pictures and photos which are much smaller than the views they represent. Research suggests that children's sense of scale develops significantly between the ages of 3 and 5 years (Frick & Newcombe 2012; DeLoache 2000). According to Huttenlocher et al. (1999), some 3-year-olds and many 4-year-olds can find objects using simple scale models and even basic plans/maps, particularly when they refer to small enclosed spaces such as a rug or sandbox. In doing this, children are using the relative position of objects at this age (Frick and Newcombe 2012) but this develops as they approach 6 to 7 years of age to using proportion to distinguish relative distances, according to Gilligan et al. (2018).

Scaling is also involved in drawing, for instance drawing a person with their body parts in proportion. Spatial scaling is important in supporting 5-7 year olds to use scaled representations in mathematics, such as a number line (Möhring et al, 2018). As spatial scaling is involved in interpreting maps, it is linked to navigation and way-finding, as explored in the next section.

Navigation and maps

Once children learn to move independently (at about 8 to 12 months, usually through crawling), they develop the ability to find objects that are hidden within reach, e.g. an object under a cloth (Campos et al., 2000). This shift in spatial search abilities later progresses to more sophisticated strategies, and by 18 months, toddlers can accurately find objects hidden in a sandbox by combining distance and landmark information (place learning) (Huttenlocher et al., 1994). However, it is not until 5 years that children begin to be able to learn and remember the turns and series of landmarks along a route (Newcombe, 2019), whilst understanding the configural structure of an area (knowledge of the spatial relationships between routes and landmarks within an environment), useful for finding shortcuts, which develops much later, between the ages of 5 and 10 years (Broadbent et al., 2014).

Maps are a representation of space. Using and drawing maps presents children with the opportunity to think about the spatial relationships between places in their environment. From about 2 and a half years, children can use a scaled model of a room to find a hidden object in the real room by using object matching (Deloache, 1989). At 3 years, children can understand a basic aerial photo of familiar objects (e.g. their toys) (Catling, 2005), and by 4 years, children can use a basic map to find a hidden object based on distance information (Huttenlocher et al., 1999) and to follow a route (Sarama & Clements, 2009), but even 5 and 6 year olds find it difficult to use maps unless they are relatively basic, and aligned with the real-world space that they represent (Spelke et al., 2001). Five year olds can, however, use an aligned map to interpret symbols on a map and to use a map to plan and navigate around a school (Sarama & Clements, 2009). At 6 years, children can locate objects and identify where they are on a map, and by 7 years children can draw a map of the area around their home from memory (Sarama & Clements, 2009). More sophisticated map use develops throughout the later primary school years. Children's initial map drawing tends to focus on the order of landmarks seen in a linear way, with ideas of relative distances and positions within an area emerging later (Liben & Yekel, 1996.)

A five year old's holiday map, showing the route from the campsite to the river.

Max and Ruth Edwards

A six year old's map of the route home from school via the park

Oli and Jenni Back

Trajectory of early learning experiences to develop spatial reasoning

The ECMG spatial reasoning trajectory provides a developmental progression (first column), how adults might sensitively support children in this phase of spatial reasoning development (second column) and how the environment might support spatial reasoning development (third column).

The trajectory is organised into approximate developmental stages but individual children may well develop spatial reasoning in an order or way that differs from the typical pathway. Statements are colour coded as broadly relating to spatial relations (in blue text) or spatial features of objects and images ('shape', in green text) to make the document easier to work with. In reality, these overlap as well as including other areas of mathematics such as measures and pattern.

Younger babies (birth to 6 months)	Children are learning to…	Adults might…	The environment might include…
	Explore space when they are free to move, roll and stretch **Develop an awareness of their own bodies**, that their body has different parts and where these are in relation to each other **Show an interest** in emptying containers	Support babies' developing awareness of their own bodies e.g. through baby massage and singing songs like This Little Piggy Went to Market. During floor play sometimes place objects that are just in or just out of reach, including small objects on cloths that babies can pull towards themselves. During water play and bathing routines, show filling and emptying different shapes and sizes of container.	Opportunities for babies to move freely in space (e.g. on carpets, grass etc.) being on the floor without objects and being free to play with their hands and feet Sensitive support for babies' play and give them long stretches of uninterrupted time to explore.
	Explore differently shaped objects and their properties through seeing and feeling/mouthing	Encourage babies' explorations of the characteristics of objects, e.g. by rolling a ball to them.	Interestingly shaped objects e.g. vegetables, spoons, corks, pinecones, balls A range of objects of various lengths and weights in treasure baskets to excite and encourage babies' interests including

	Respond to size, reacting to very big or very small items that they see or try to pick up		larger and smaller items, e.g. a larger and a smaller soft toy.
Older babies (6 to 12 months)	**Children are learning to…**	**Adults might…**	**The environment might include…**
	Engage with positions and directions, using gestures and concepts like *'in', 'on', 'under', 'up', 'down'* sometimes moving objects or pointing to where they would like to go. **Enjoy hiding** and finding with themselves and objects **Begin to put objects** inside others and take them out again **Explore space** by crawling and walking **Show an interest** in objects which are the same in contrasting sizes e.g. selecting a big spade or a small spade.	Use spatial words during everyday play and routines, e.g. when sweeping leaves *off* a path or water *down* the drain. Demonstrate rolling a ball or moving objects over shorter and longer distances. Play peekaboo games Support babies' embodied understanding of position, e.g. singing songs using positional language such as *The Grand Old Duke of York* or taking them on a laundry basket ride and saying '*Up, up, up!*' as you sweep them up into the air, and '*Down, down, down!'* as you come down, maybe making your voice go up and down too. Play games that involve curling and stretching, popping *up* and bobbing *down*. When sharing picture books, take opportunities to point out differences in size, e.g. a big truck and a little truck, or a big cat and a small kitten. Talk about the properties of shapes, e.g. *flat, round, curvy, bumpy.*	Books with opportunities for using spatial language, e.g. *Where's Spot?* by Eric Hill and *Peepo* by Janet and Allan Ahlberg and use opportunities in all other books to use spatial words Resources on different levels and at differing heights and talk about these, e.g. '*There's your teddy **up** on the shelf*'. Bags, boxes and cloths for items to be stored, hidden and transported. Nested boxes, cups or toys, i.e. boxes/cups/toys of different sizes that fit inside each other Books about bodily awareness and movement, e.g. *More, More, More said the Baby* by Vera B Williams Low mirrors to support babies to develop bodily awareness. Objects demonstrating marked differences in size e.g. dolls and adult chairs, tiny and big bears, blocks and containers and talk about *big* and *small* Blocks and boxes to build with

	Respond to changes of shape, e.g. flattening mud pies **Attempt to fit shapes** into spaces on inset boards, sometimes successfully	Talk about simple properties of objects such as *big* and *small*, *long* and *short*, *high* and *low*, etc. during play and everyday contexts, e.g. when out and about, on swings/see-saw/slides, toy play, chopping food. Talk about and show the shape of objects can be changed, e.g. a sponge can be squeezed or stretched into a different shape Demonstrate putting a smaller item inside a similarly shaped larger item (e.g. smaller bowl inside a larger bowl)	objects that stack e.g. wooden blocks, stacking cups Malleable materials where children can change the shape and size e.g. mud, playdough, enlarging a puddle, chop an apple. Shape sorters, posting toys and inset board puzzles for children to explore independently as well as co-operatively with adults (e.g. posting pompoms through a cardboard tube or hole in a plastic lid). Bags/boxes to fit things inside and to transport around the environment A range of containers for water play.
Toddlers (1-2 year olds)	**Children are learning to…**	**Adults might…**	**The environment might include…**
	Begin to use gestures and perhaps words for *in, on, under, up, down* as instructions **Enjoy filling and emptying** containers. **Investigate fitting themselves** inside and moving through spaces **Push objects through** holes, moving them around to find the hole	Use 'tidy up time' to promote logic and reasoning about where things fit in or are kept. Regularly use gestures in familiar contexts alongside spatial language e.g. pat the cushion when asking a child to sit beside you. Support children's interest in body-sized spaces by providing suitable boxes etc and	Specific places or spaces for items to be stored and fitted into for tidying. Children's books about fitting inside boxes. Boxes, outside spaces and furniture to get inside and move through. Shape sorters and other toys where items can be hidden, enclosed or posted through holes.

	Begin to explore stacking objects with flat surfaces together, e.g. stacking blocks and cups **Explore familiar environments**, moving freely around and enjoying finding out about the world from the new viewpoints they experience **Show an interest in shape and size,** sometimes responding to words or gestures for *big* and *small, round* or *flat* **Attempt to fit shapes** into spaces on inset boards or puzzles, beginning to select a shape for a specific space and put objects of similar shape inside each other **Use blocks to create** their own simple structures and arrangements including lines of identical shapes	provide commentary on going '*inside*', *'through', 'under', 'over' and 'between'.* Build towers *'up'* for the child to knock *'down'* Hide a favourite toy '*under'* a container or cloths. Value children exploration of their environment indoors and outdoors Talk about the properties and size of shapes (e.g. flat, round, bumpy, big, small) when selecting them to fit into spaces, e.g. *Oh dear, the one with corners won't fit, we need a round one.* Play alongside children building their own structures, building your own structures and providing a commentary or building together. Talk about size in everyday play and routines, extending the range of vocabulary heard e.g. *bigger/smaller than, little bit bigger than, further, nearer.* Comment on children's selection of big objects and attempts to move them.	Access to small spaces where children like to hide, squeeze into to fit through. Larger spaces with a variety of levels to give a range of viewpoints. A range of inset board and puzzles with pieces. A range of construction materials, e.g. wooden blocks, packaging. Storage with photos to show where things are kept. Objects of similar shapes that can nest inside each other, e.g. pots, boxes, baskets A range of objects, including big, heavy and awkward ones that can be transported, both indoors and outdoors.

2 year olds	Children are learning to…	Adults might…	The environment might include…
	Respond to position and direction words to identify location, e.g. *in, out, on, up, down, over there, long way away* Use **position and distance** to identify the location of objects in an enclosed space **Maneuver toys** and themselves around objects and the environment. **Place objects with both hands,** creating patterns and constructions with two sides which match **Explore what can be seen and how things look** from different viewpoints, e.g. partially hidden, looking between your legs or hanging upside down from a sofa **Order** objects by size **Find their way** around familiar environments, e.g. the way to the toilet/sand tray or to park the ride-on toy outdoors	Demonstrate arranging things, emphasising position and direction language, e.g. setting the table in the home corner or lining up cars to roll down the slope. Play games involving jumping, running and hiding and model making very simple obstacle courses. Model making things with matching components on two sides, sometimes reflected Play hide-and-reveal games with objects in boxes and under cups. Look for opportunities to fit objects according to their size, e.g. whether a teddy will fit in a bed. Support children to order things e.g. stacking all the cups in a stacking-cup set, all the nesting dolls. Help children to create simple roads and rail tracks and talk about position, e.g. *Shall we put this piece next to the bridge or the river?*	Spaces for children to hide, travel *through, over, down* and *around*. Books that include fitting into spaces, e.g. lift the flap and *What will Fit?* by Grace Lin. Sand trays with sufficient sand and objects which can be buried. Similar items and toys of different sizes such as dolls, trucks, bottles, cups, boxes or spoons. Large floor level mirrors. Small world play provides an opportunity to look 'down' on a world and to think about different perspectives. Wheelbarrows, bags, baskets and flexi tubs to enable children's fascination with transporting.
	Respond to differences between shapes and sizes, and associated	Talk about size and shape properties using informal language and gesture, e.g. *flat, round, curvy, corner, pointy.*	Inset board and jigsaw puzzles of increasing complexity.

	informal language and gesture (e.g. *flat, round, curvy, corner, giant, teeny*) **Recognise that two objects** have the same shape, e.g. chooses two circles for eyes **Predict and fit pieces** into inset puzzles **Make simple constructions** with blocks, combining identical shapes to make walls, towers, etc.	Demonstrate the language of size and distance to describe everyday items and contexts, e.g. *huge, much smaller, longer, taller, shorter, long way away.* Demonstrate choosing a particular shaped item for a purpose, e.g. a *pointy* carrot for a nose Demonstrate comparing two objects to see if they have the same shape, e.g. two blocks or collage pieces/stickers. Talk about the shape of the pieces and the holes when fitting pieces into inset puzzles. When building, talk about the shape of the blocks you are selecting and why.	A variety of construction materials for indoor and outdoor play.
3 year olds	Children are learning to…	Adults might…	The environment might include…
	Respond to and use position and direction words, e.g. *inside, under, next to, over, through, along, upside down* Use **relative position and distance** to identify the location of objects **Move and rotate** objects to fit the space or create the shape they would like Make patterns with some **symmetrical** elements, often by placing objects on	Demonstrate the language for position and direction in everyday interactions, accompanying these with gestures. Find out and use equivalent terms for these in children's home languages and Makaton. When tidying, encourage children to look for and retrieve out of place items. Play together with small world toys for children to create their own environments, discussing where they want to position items and the reasons for these. Make a small world model the same as theirs,	Games involving children positioning themselves *inside, on top, underneath* Trails and treasure hunts, e.g. using recordings of verbal instructions (using talking pegs, tins, microphones, postcards etc), e.g. *'Look **under** the bench'.* Obstacle courses and materials to create these, so children go *over*, *through* and *between* Books such as *Up and Down* by Britta Teckentrup and *Inside, Outside, Upside*

3 year olds	the other side to 'match' and perhaps some that grow from the middle outwards (**radiating patterns**) **Perspective-take,** recognise objects that are near or far away. **Recognise** things represented by scaled toys and small world environments (such as dinosaurs, cars, figures, dolls house, farms) **Find their way** around familiar environment. **Recognise and predict** familiar routes e.g. says *garage* before you see it	copying each move they make with a commentary. Demonstrate moving and turning jigsaw pieces to check if they will fit. Discuss how reflections in mirrors and ponds etc. are the other way round, or upside down Discuss patterns and natural objects with reflective or rotational symmetry When looking out of the window, in pictures or on walks, point out things or people that are near or a long way away and how they appear larger or smaller. Create walkways together, e.g. stepping stones, hollow blocks, planks, chalk lines, log slices.	*Down* by Jan and Stan Berenstain to stimulate discussion about position and direction. Materials to explore small world play and freely create rail tracks and road layouts Mirrors to explore and play with Outings to look at reflections in puddles, ponds or rivers, taking photographs. Shadow silhouettes or specific places and containers for children to tidy up items by fitting them into the designated space. Photographs of things and familiar places from different positions and perspectives. Indoor and outdoor spaces, stimulating children make their own choices and create routes, e.g. with wheeled toys.
	Show awareness of similarities and differences between shapes, including selecting items by their shape and size so they are appropriate (e.g. chooses a puzzle piece by its shape, chooses a triangular block for a roof and the wedge shaped block for a ramp). Respond to informal shape language (e.g. *straight, round, slanting, pointy*)	Draw children's attention to shapes in the environment and describe them using informal language, common shape names and gestures. Discuss 'nearly' shapes (e.g. *This is almost a square but it's got curvy corners).* Find out and use equivalent terms for shapes in home languages and Makaton.	Resources with different shape properties to handle, move around and explore e.g. packaging for box modelling, pattern blocks. Food items cut into different shapes, e.g. sandwiches, carrots cut into sticks or circles. Freely explore playdough with knives, paper with scissors.

	and common shape names (e.g. *circle*, *triangle*) **Partition and combine** shapes to make new shapes with 2D and 3D shapes (e.g. cutting 'square' sandwiches into different shapes, putting blocks together to make a 'floor') **Create** arches and enclosures when building, using trial and improvement to select blocks	Encourage children to select blocks for specific purposes when building, e.g. *What will we use for the elephants trunk?* Offer an appropriate or inappropriate shape for what you think the child's purpose might be (to investigate their thinking). Value children's constructions (e.g. helping to display them or taking photos of them) and talk about how the shapes have combined to make new shapes. Sensitively support and challenge experienced builders to make specific structures e.g. bridges and rooms. Offer choices (*Would you like one of these or one of these next*?).	Lightboxes for silhouette play. Books and props for traditional tales involving ordering and size, e.g. *The Three Billy Goats Gruff, Goldilocks* and *The Enormous Turnip.* Large and small blocks and boxes available for construction both indoors and outdoors, e.g. for making entrances, bridges, walls and dens.
4 and 5 year olds	**Children are learning to…**	**Adult interactions**	**Provision**
	Understand relative position, such as *between, in front of, behind, before* and *after* (where the position is in relation to other things, e.g. *in front* of the house or *behind* the wall) **Follow and give directions,** e.g. *forwards, backwards, sideways,* and *left* and *right* turns when accompanied by gestures **Solve problems** (e.g. *Will it fit?)* involving comparisons and predictions	In everyday play and routines, encourage children to describe position and give directions, e.g. in small world play, when following pathways or creating obstacle courses. Play 'barrier games' where you give instructions to a partner to 'make it the same', with an identical set of objects. Begin without a barrier (copying) then introduce one when they become proficient.	Controllable and programmable toys, with simple routes and obstacles to negotiate. Small mirrors for exploring reflection. Provide toys, pictures and pen/paper for experimentation. Toys or packaging to create marble runs, predicting the path of the marble/ball and solving problems in the marble run design.

4 and 5 year olds	*about* length/distance, volume/ capacity; paying attention to fairness and accuracy e.g. matching ends and 'fullness' ***Turn* and *flip* objects** in order to create models and make shapes fit, visualising and predicting how they will look, including to create a **mirror image** (sometimes doing it the wrong way) Create **reflections** with a vertical axis (top to bottom), or with four lines of symmetry (sometimes repeating rather than reflecting) Making **radiating patterns** (grown from the centre) with reflective and rotational symmetry **Explore what can be seen from different viewpoints,** e.g. knows how to hide effectively from a 'seeker'; compares what they can see e.g. from the top and bottom of the climbing frame. **Engage with 3D models & 2D map-making** of familiar environments, sequencing landmarks and designing small worlds, e.g. a playground in a builder's tray and rail tracks that join up.	Look out for everyday opportunities to make comparisons, e.g. predicting if the tray will fit in the role-play oven then trying it. Engage in solving problems such as: Which car will roll furthest? (predict where the car will stop), find a stick exactly as long as your arm/little finger/leg or which jug will hold enough water for everyone to have a cupful? Challenge children to make as many different shapes as they can from 4 or 5 multilink cubes. Discuss whether any are the same but *the other way around* (mirror images). Encourage children to turn and flip objects to solve problems such as selecting the correct pieces so that a train track joins up or to make a marble run that works (and is stable). Model strategies for solving shape puzzles, e.g. hovering a piece over a gap and turning shapes to see if they will fit, then doing this in their head. Model flipping shapes to match a mirror image Children's face drawings cut in half: complete the whole face- compare with a mirror and discuss	Crates, tyres, planks, canes/sticks, string and logs for children to create their own obstacle courses and dens. Include clipboards for children to record and make alternative designs. Mirrors, including hinged mirrors Books involving symmetry such as 'Make a bigger puddle, make a smaller worm' by Marion Walter Pattern outlines with reflective symmetry to fill with pattern blocks and shapes 'half patterns', to complete with pattern blocks or pegboards, large squared paper and tiles. Provide mirrors to check symmetry. Free play and outline puzzles with a range of shapes, including pattern blocks, mosaic tiles and Numicon baseboards or tray surrounds. Photographs of the children's models taken from different viewpoints, e.g. aerial and side view of the same block model. Engage families in taking photos of familiar things from different viewpoints. Small world play, train and road layouts, miniature gardens in trays, for children to create, arrange and describe.

4 and 5 year olds	Notices landmarks and uses these to **find their way** around familiar places.	Play partner mirrors: one child makes a shape or movement and the other mirrors it Discuss what might be seen using small world scenarios or asking '*What might this be?*' with silhouettes and photos from different viewpoints and including partial views. Encourage children to create scaled down versions of familiar places, e.g. their bedroom in a shoebox or a small world version of the local park. In addition to free play, challenge children to make a model from a 2D picture. Encourage children to focus closely. Discuss *what is the same* and *what is different* between their model and the original. Make simple line-maps on a blank piece of paper, drawing arrows to show direction and modelling the language as you draw it. Discuss the local environment and visit local places, examining photographs and simple maps. Encourage children to recall the order of landmarks on familiar routes around their local environment.	Photos of familiar places to inspire model making, painting/drawing, block play and small world play. Online maps with the children to look at routes, landmarks and homes on 'street view' and discuss what can be seen next to, in front of, behind, opposite, etc. Story books about journeys e.g. *Rosie's Walk* and *Changes, Changes* by Pat Hutchins Rolls of wallpaper on the floor for children to freely draw their own road layouts and maps, with toy cars and people or maps related to familiar stories. Photo books and videos of familiar routes and landmarks to stimulate conversation using relative language, e.g. *'in front of'*, *'behind'*, and *'next'*.

4 and 5 year olds	**Understand and use mathematical** terms to describe shapes (e.g. *cylinder*, *cone*, *square*) and properties (e.g. *straight*, *curved*, *edges, corners*) as well as informal language and analogies (e.g. slanty, wiggly, box or roof-shaped) **Identify several examples** of the same shape (e.g. different kinds of triangles) and recognise that a shape is the same even in different orientations (e.g. turned round) **Solve shape puzzles** of increasing complexity, selecting shapes according to their properties **Compose and decompose** shapes, knowing how shapes combine to make other shapes, (e.g. triangles making a rectangle) and identifying shapes within shapes (decomposing) **Build complex compositions** including repeated units, (such as arches made of three blocks), corners (pieces at right angles) and ramps. Selects shapes to solve a problem **Plan mentally by** visualising what they will build and selecting blocks needed	Play games focusing on the properties of shapes, e.g. hiding and partially revealing a shape, asking children to say what different shapes it could be or could not be and why, or using a feely-bag to identify familiar items as well as 3D shapes. Discuss different examples of shapes (e.g. different types of triangle such as equilateral and right-angled) in a variety of orientations (e.g. squares positioned on a corner). Discuss the shapes that emerge when children paint, draw and collage or that they notice in the environment. Discuss which shapes make other shapes, e.g. triangles making rectangles and hexagons with pattern blocks or mosaic tiles. Teach strategies for solving shape and jigsaw puzzles, e.g. describing shape properties and modelling the mathematical vocabulary, such as *straight, corner, edges.* Challenge children to make more complex constructions (perhaps in story contexts), e.g. with towers or arches, a window or a staircase.	Books e.g. *The Smartest Giant in Town* by Julia Donaldson, *Big Blue Whale* by Nicola Davies and Nick Malan. *Is it larger, Is it smaller?* by Tala Hoban, as well as adapting familiar stories to have a shape theme (e.g. We're going on a *square* hunt) A wide range of materials for construction indoors and outdoors including unit blocks and a range of recycled materials which provide real life examples of shapes e.g. kitchen roll tubes, cube tissue boxes, party hats, tyres, drainpipes, planks, canes and connectors etc. A wide range of resources for shape play including pattern blocks and mosaic tiles. Shape and jigsaw puzzles with different levels of challenge. Old greeting cards to be cut up for children to make into jigsaws. Photos of shapes in nature and manufactured items as well as buildings from around the world and local landmarks for children to construct and draw the shapes they see. Books that include shapes in the environment. Printing using a variety of 3D items. Discussing the 2D printed shapes they make.

			Measuring cylinders/beakers in the water area and shadow these so children are ordering by size at tidy up time.
6 and 7 year olds	**Children are learning to…**	**Adults might…**	**The environment might include…**
	Understand spatial concepts and use the language of: **position** e.g. *before, after, between, opposite, overlapping* **direction** e.g. *left* and *right (describing turns that are more/less than 90 degrees), diagonally, across* and **orientation** e.g. *upside down, back to front, slanting* Predict the path of travelling objects, in terms of **distance and direction**. Solve **shape puzzles** of increasing complexity, predicting which shapes will fit and how; create own puzzles. **Build complex constructions** including repeated units, staircases and ceilings. **Visualise transformations** by sliding and reflecting objects, rotating half and quarter turns; predicting how they will look. Reflect images or patterns over a horizontal axis (and sometimes diagonal)	Use a range of language to describe the location of objects and relevant landmarks when exploring familiar or unfamiliar environments Briefly show children a simple multilink or Lego model and ask them to build it from memory. Reveal and discuss similarities and differences using spatial language). Build children's physical and spatial co-ordination by playing ball games, rolling games and experimenting with vehicles and ramps. Using pentominoes, find different shapes with 5 squares (whole sides touching), prompting children to discuss which are mirror images or rotations of others Encourage children to predict the shape of the hole when folding and cutting paper. Cut a bit out of a folded piece of paper and ask children to justify their prediction before unfolding Describe a simple model that is out of sight. Imagine turning it upside down or	'Barrier games' with increasingly sophisticated pieces; e.g. blocks of the same colour, pattern blocks, paper-tangrams. Materials for creating interesting small world routes for cars and trains, recreating routes and journeys from stories and obstacle courses outdoors Designing plans and maps for these. Programmable toys to direct through obstacle courses or to follow routes. Children can play robots and direct each other to follow routes with landmarks. Photographs of familiar items or their own models, taken from a range of perspectives. Mirrors and half images to complete (drawing). Play symmetry games with a partner (see barrier games in our Firm Foundations guidance for 5-7s). Sheets of paper quartered, for children to draw patterns reflected vertically and horizontally. Provide long strips of paper

6 and 7 year olds	Interpret and predict what and how things will appear from different viewpoints **(perspective–taking)**, including when partially obscured or from above (plan view). Interpret and make 3D models & simple 2D maps/plans of familiar environments, identifying the representation of the real world feature. When drawing maps or familiar routes, place things at approximately correct relative distances e.g. near my home **Begin to use proportional language** e.g. halfway, middle **Navigate simple routes.** Plan a simple route in a familiar environment using landmarks	what it might look from the back or top. Show the actual model, view it from different perspectives and discuss how it looked the same or different in their head. Support children to build more complex constructions, using exploded model diagrams, e.g.  Encourage them to notice smaller units of combined shapes within models. Encourage children to create diagrammatic instructions, with drawing or writing, for others to make a model. Construct Lego marble mazes / roadways together, discuss left and right, forwards and backwards. Encourage problem solving. Create a classroom, school or playground map and give directions (referencing landmarks along the way) to find specific places or hidden items.	to make zig-zag folds and cut out people shapes holding hands (paper dolls). Mirror puzzle books such as 'The magic mirror book' by Marion Walter. Resources and examples for making paper snowflakes: paper folded in half, then in three, to cut out designs on the fold Images of constructions made with blocks (including exploded models) for children to discuss, compare and improve upon. Consider a 'Lego club' with family members or older children. Clipboards and pens for children to draw their models and design new ones. Plan views (or oblique views which are not quite above) of environments (e.g. classroom). Perhaps, use paper maps for role-play (e.g. travel agents) and Google maps for aerial photographs to identify familiar routes viewing them from above e.g. from school to the park or shops, from home to school. Plenty of opportunities to practise and develop confidence in playing bat and ball, over varying distances. Play at rolling balls down ramps and catching it, encourage children to invent their own anticipatory games.

6 and 7 year olds		Encouraging children to problem solve involving scale, making a model skeleton that is half your size (in proportion) or work out how large a giant would be from their footprint, for example. Compare different approaches. Draw maps of familiar places and routes and discuss the relative distances between landmarks. Encourage children to make maps for other children (e.g. to find hidden objects). Briefly show children a simple model and ask them to build it from memory (given a selection of shapes). Reveal and discuss similarities and differences.	Small world play to re-create familiar routes and discuss the relative positions of landmarks and distances between landmarks.
	Use mathematical terms to describe regular and irregular shapes (e.g. *cuboid, prism, pyramid, hexagon, octagon*). Describe shapes using mathematical terms for properties (e.g. *faces, diagonals, right angles, wide, narrow*). Use informal language, gesture and analogies (e.g. zigzag, bumpy, wedge-shaped). **Identify extreme and non-examples** of the same shape, e.g. plastic 'rectangles' as cuboids, but not rectangular-ish shapes with rounded corners (e.g. mobile phone);	Model folding a sheet of paper in half and making one straight cut, unfolding to see how many sides the shape has when unfolded. Place a collection of 3D shapes into a feely-bag to identify and match with some they can see. Predict what cross-sections, e.g. of fruit, will look like, including when cut horizontally and vertically. Predict 3D shapes from nets and vice versa (e.g. Polydron). Solve problems such as which of these nets will make a cube.	A range of boxes and cartons to de-construct (into nets) and re-construct or turn inside out. Provide 3D shape-making resources, like Polydron or Clixi, or large scale outdoor materials. Paper and card to fold and cut shapes, e.g. snowflakes. A range of jigsaws of different complexity; consider a 'jigsaw club' with family members or older children.

6 and 7 year olds	mathematically similar shapes of different sizes. **Decompose shapes** in different ways e.g. predicting folds, nets and cross – sections. Relate **2D and 3D** in making models from photos and plans (2D-3D) and do drawings of 3D models and arrangements (3D-2D).	Use a construction resource such as Geostrips to make shapes, discuss different angles and the properties of shapes when transformed (e.g. squashed).	

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